VTU QUESTION PAPER SOLUTION

June 2012

1. List any three situations when simulation tool is appropriate and not appropriate tool. 6 M

Ans: When Simulation is the Appropriate Tool
- Simulation enables the study of and experimentation with the internal interactions of a complex system, or of a subsystem within a complex system.
- Informational, organizational and environmental changes can be simulated and the effect of those alternations on the model’s behavior can be observer.
- The knowledge gained in designing a simulation model can be of great value toward suggesting improvement in the system under investigation
- By changing simulation inputs and observing the resulting outputs, valuable insight may be obtained into which variables are most important and how variables interact

When Simulation is Not Appropriate
- Simulation should be used when the problem cannot be solved using common sense.
- Simulation should not be used if the problem can be solved analytically.
- Simulation should not be used, if it is easier to perform direct experiments.
- Simulation should not be used, if the costs exceeds savings.

b. Define the following terms used in simulation 6 M

Ans: i) discrete system
Systems in which the changes are predominantly discontinuous are called discrete systems. Ex: Bank – the number of customer’s changes only when a customer arrives or when the service provided a customer is completed.

![Diagram](image)

ii) continuous system
Systems in which the changes are predominantly smooth are called continuous system. Ex: Head of a water behind a dam.
iii) stochastic system
Has one or more random variable as inputs. Random inputs leads to random outputs. Ex: Simulation of a bank involves random interarrival and service times.

iv) deterministic system
Contains no random variables. They have a known set of inputs which will result in a unique set of outputs. Ex: Arrival of patients to the Dentist at the scheduled appointment time.

v) entity
An entity is an object of interest in a system. Ex: In the factory system, departments, orders, parts and products are the entities.

vi) attribute
An attribute denotes the property of an entity. Ex: Quantities for each order, type of part, or number of machines in a Department are attributes of factory system.

c. Draw the flowchart of steps involved in simulation study. 8 M

2a) Consider the grocery store with one check out counter. Prepare the simulation table for eight customers and find out average waiting time of customer in queue, idle time of server and average service time. The inter arrival time (IAT) and service time (ST) are given in minutes.
IAT : 3,2,6,4,4,5,8  
ST(min) : 3,5,5,8,4,6,2,3

Assume first customer arrives at time t=0

<table>
<thead>
<tr>
<th>Clock</th>
<th>LQ(t)</th>
<th>LS(t)</th>
<th>FEL</th>
<th>Comment</th>
<th>B</th>
<th>MQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>(D,4)</td>
<td>First A occurs (a* = 8) schedule next A (s* = 4) schedule next D</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>(A,8)</td>
<td>First D occurs (D,4)</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>1</td>
<td>(D,9)</td>
<td>Second A occurs (A,8) schedule next A (s* = 1) schedule next D</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>(A,14)</td>
<td>Second D occurs (D,9)</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

2b) Suppose the maximum inventory level is M 11 units and the review period is 5 days estimate by simulation the average ending units in inventory and number of days when a shortage condition occurs. Initial simulation is started with level of 3 units and an order of 8 units scheduled to arrive in two days time. Simulate for three cycles (15 days) The probability for daily demand and lead time is given below

<table>
<thead>
<tr>
<th>Demand</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.1</td>
<td>0.25</td>
<td>0.35</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lead time</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

RD for demand: 24,35,65,25,8,85,77,68,28,5,92,55,49,69,70
3a) Define the term used in discrete event simulation: 6M

i) System safe
A collection of variables that contain all the information necessary to describe the system at any time

ii) list
A collection of (permanently or temporarily) associated entities ordered in some logical fashion (such as all customers currently in a waiting line, ordered by first come, first served, or by priority).

iii) event
An instantaneous occurrence that changes the state of a system as an arrival of a new customer).

iv) delay
An instantaneous occurrence that changes the state of a system as an arrival of a new customer).

v) system
A collection of entities (e.g., people and machines) that ii together over time to accomplish one or more goals.

3b) Six dump trucks are used to haul coal from the entrance of a small mine to railload. Each truck is loaded by one of two loaders. After loading truck moves to scale, to be weighed. After weighing a truck begins to travel time and then returns to loader queue. It has been assumed
that five of trucks are at loader and one at scale at time 0. By using event scheduling algorithm find out busy time of loader and scale and stopping time $E$ is 64 mins.

<table>
<thead>
<tr>
<th>Loading time</th>
<th>10</th>
<th>5</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>10</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighing time</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>16</td>
<td>12</td>
<td>16</td>
<td>--</td>
</tr>
<tr>
<td>Travel time</td>
<td>60</td>
<td>100</td>
<td>40</td>
<td>40</td>
<td>80</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Clock t</th>
<th>LQ(t)</th>
<th>L(t)</th>
<th>WQ(t)</th>
<th>W(t)</th>
<th>Loader queue</th>
<th>Weigh queue</th>
<th>FEL</th>
<th>BL</th>
<th>BS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>DT4 DT5 DT6 (EL,5,DT3) (EL,10,DT2) (EL,12,DT1)</td>
<td></td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>DT5 DT6 DT3 (EL,10,DT2) (EL,5+5,DT4) (EW,12,DT1)</td>
<td></td>
<td>10</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>DT6 DT3 DT2 (EL,10,DT4) (EW,12,DT1) (EL,10+10,DT5)</td>
<td>20</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>DT3 DT2 DT4 (EW,12,DT1) (EL,20,DT5) (EL,10+15,DT6)</td>
<td>20</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>DT2 DT4 (EL,20,DT5) (EW,12+12,DT3) (EL,25,DT6) (ALQ,12+60,DT1)</td>
<td>24</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>DT2 DT4 DT5 (EW,24,DT3) (EL,25,DT6) (ALQ,72,DT1)</td>
<td>40</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>DT4 DT5 (EL,25,DT6) (EW,24+12,DT2) (ALQ,72,DT1) (ALQ,24+100,DT3)</td>
<td>44</td>
<td>24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average Loader Utilization = \( \frac{44}{2} = 0.92 \)

Average Scale Utilization = \( \frac{24}{24} = 1.00 \)

4a) the number of Hurricanes hitting the coast of Indian follows poisson distribution with mean \( \alpha = 0.8 \) per year Determine:

i) The probability of more than two hurricanes in a year

ii) The probability of more than one hurricane in a year 6 M
4b) Explain the terms used in queuing notations of the form A/B/C/N/K

Ans: A notation system for parallel server queues: A/B/c/N/K
- A represents the interarrival-time distribution,
- B represents the service-time distribution,
- c represents the number of parallel servers,
- N represents the system capacity,
- K represents the size of the calling population.

4c) List the steady state parameters of M/G/1

- Single-server queues with Poisson arrivals & unlimited capacity.
- Suppose service times have mean \( \frac{l}{m} \) and variance \( s^2 \) and \( r = \frac{l}{m} < 1 \), the steady-state parameters of M/G/1 queue:

\[
\begin{align*}
\rho &= \frac{\lambda}{\mu}, \quad P_0 = 1 - \rho \\
L &= \rho + \frac{\rho^2 (1 + \sigma^2 \mu^2)}{2(1 - \rho)}, \quad L_0 = \frac{\rho^2 (1 + \sigma^2 \mu^2)}{2(1 - \rho)} \\
W &= \frac{1}{\mu} \frac{\lambda (1 + \mu^2 + \sigma^2)}{2(1 - \rho)}, \quad W_0 = \frac{1}{\mu} \frac{\lambda (1 + \mu^2 + \sigma^2)}{2(1 - \rho)} \\
Q &= \frac{W}{\rho}, \quad Q_0 = \frac{W_0}{\rho}
\end{align*}
\]

5a) Using multiplicative congruential method, generate random numbers to complete cycle. Explain maximum density and maximum period, \( A=11, M=16, X_0=7 \)

The sequence of \( X_i \) and subsequent \( R_i \) values is computed as follows:

\( X_0 = 27 \)
\( X_1 = (17 \cdot 27 + 43) \mod 100 = 502 \mod 100 = 2 \)
\( R_1=2/100=0.02 \)
\( X_2 = (17 \cdot 2 + 43) \mod 100 = 77 \mod 100 = 77 \)
\( R_2=77/100=0.77 \)
\( X_3 = (17 \cdot 77 + 43) \mod 100 = 1352 \mod 100 = 52 \)
\( R_3=52/100=0.52 \)

5b) Using suitable frequency test find out whether the random numbers generated are uniformly distributed on the interval \([0,1]\) can be rejected. Assume \( \alpha=0.05 \) and \( D_\alpha=0.565 \). The random numbers are 0.54, 0.73, 0.98, 0.11, 0.68
Ans: First, the numbers must be ranked from smallest to largest. The calculations can be facilitated by use of Table 7.2. The top row lists the numbers from smallest (R(1) ) to largest (R(n) ) .The computations for D+, namely i /N -R(i) and for D-, namely R(i) - ( i - 1 ) / N, are easily accomplished using Table 7.2. The statistics are computed as D+ = 0.26 and D- = 0.21. Therefore, D = max{0.26, 0.21} = 0.26. The critical value of D, obtained from Table A.8 for a = 0.05 and N = 5, is 0.565. Since the computed value, 0.26, is less than the tabulated critical value, 0.565, the hypothesis of no difference between the distribution of the generated numbers and the uniform distribution is not rejected.

6a) Develop a random variate generator for X with pdf given below

\[ F(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases} \]

6b) Explain with an example, importance of data distribution using histogram.

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_i )</td>
<td>0.1306</td>
<td>0.0422</td>
<td>0.6597</td>
<td>0.7965</td>
<td>0.7696</td>
</tr>
<tr>
<td>( X_i )</td>
<td>0.1400</td>
<td>0.0431</td>
<td>1.078</td>
<td>1.592</td>
<td>1.468</td>
</tr>
</tbody>
</table>

A frequency distribution or histogram is useful in identifying the shape of a distribution.

A histogram is constructed as follows:
1. Divide the range of the data into intervals (intervals are usually of equal width; however, unequal widths however, unequal width may be used if the heights of the frequencies are adjusted).

2. Label the horizontal axis to conform to the intervals selected.

3. Determine the frequency of occurrences within each interval.

4. Label the vertical axis so that the total occurrences can be plotted for each interval.

5. Plot the frequencies on the vertical axis.

- If the intervals are too wide, the histogram will be coarse, or blocky, and its shape and other details will not show well. If the intervals are too narrow, the histogram will be ragged and will not smooth the data.
- The histogram for continuous data corresponds to the probability density function of a theoretical distribution.

6c) The following is set of single digit numbers from a random number generator. Using appropriate test, check whether the numbers are uniformly. \( N = 50, \alpha = 0.05 \) and \( X^2_{0.05.9} = 16.9 \)

\[
6, 7, 0, 6, 9, 9, 0, 6, 4, 6, 4, 0, 8, 2, 6, 6, 1, 2, 6, 8, 5, 6, 0, 4, 7 \]

\[
1, 3, 5, 0, 7, 1, 4, 9, 8, 6, 0, 8, 6, 6, 7, 1, 0, 4, 7, 9, 2, 0, 1, 4, 8
\]

7a) Records pertaining to the monthly number of job related injuries at an underground coal mine were being studied by federal agency. The values of past 100 months are as follows:

<table>
<thead>
<tr>
<th>Injuries/month</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of occurrence</td>
<td>35</td>
<td>40</td>
<td>13</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Apply the chi square test to these data to test the hypothesis that the distribution is poisson with mean 1.0 and \( \alpha = 0.05 \) and \( X^2_{0.05.9} = 7.81 \).

The first few numbers generated are as follows:

\[
X^1 = 7^5 \text{(123,457)} \mod (2^{31} - 1) = 2,074,941,799 \mod (2^{31} - 1)
\]

\[
X^1 = 2,074,941,799
\]

\[
R^1 = X^1 \div 2^{31}
\]

\[
X_2 = 7^5 (2,074,941,799) \mod (2^{31} - 1) = 559,872,160
\]

\[
R_2 = X_2 \div 2^{31} = 0.2607
\]

\[
X_3 = 7^5 (559,872,160) \mod (2^{31} - 1) = 1,645,535,613
\]

\[
R_3 = X_3 \div 2^{31} = 0.7662
\]
7b) Differentiate between terminating and steady state simulation with respect to output analysis with an example. 10 M

Confidence-Interval Estimation

- Prediction Interval (PI):
  - A measure of risk.
  - A good guess for the average cycle time on a particular day is our estimator but it is unlikely to be exactly right.
  - PI is designed to be wide enough to contain the actual average cycle time on any particular day with high probability.
  - Normal-theory prediction interval:
    \[
    Y_n \pm t_{a/2,R-1} S \sqrt{1 + \frac{1}{R}}
    \]
  - The length of PI will not go to 0 as R increases because we can never simulate away risk.
  - PI’s limit is: \[\theta \pm z_{a/2} \sigma\]

7.4 Output Analysis for Terminating Simulations

- A terminating simulation: runs over a simulated time interval \([0, T_E]\).

A common goal is to estimate:

\[
\theta = E\left(\frac{1}{n} \sum_{i=1}^{n} Y_i\right), \quad \text{for discrete output}
\]
\[
\phi = E\left(\frac{1}{T_E} \int_{0}^{T_E} Y(t) dt\right), \quad \text{for continuous output } Y(t), 0 \leq t \leq T_E
\]

- In general, independent replications are used, each run using a different random number stream and independently chosen initial conditions.

8a) Explain with a neat diagram verification of simulation model. 10 M
Verification of Simulation Models

- The purpose of model verification is to assure that the conceptual model is reflected accurately in the computerized representation.

- The conceptual model quite often involves some degree of abstraction about system operations, or some amount of simplification of actual operations.

Many suggestions can be given for use in the verification process:
1. Have the computerized representation checked by someone other than its developer.
2. Make a flow diagram which includes each logically possible action a system can take when an event occurs, and follow the model logic for each action for each event type.
3. Closely examine the model output for reasonableness under a variety of settings of Input parameters.
4. Have the computerized representation print the input parameters at the end of the Simulation to be sure that these parameter values have not been changed inadvertently.
5. Make the computerized representation of self-documenting as possible.
6. If the computerized representation is animated, verify that what is seen in the animation imitates the actual system.
7. The interactive run controller (IRC) or debugger is an essential component of Successful simulation model building. Even the best of simulation analysts makes mistakes or commits logical errors when building a model. The IRC assists in finding and correcting those errors in the follow ways:
   (a) The simulation can be monitored as it progresses.
   (b) Attention can be focused on a particular line of logic or multiple lines of logic that constitute a procedure or a particular entity.
   (c) Values of selected model components can be observed. When the simulation has paused, the current value or status of variables, attributes, queues, resources, counters, etc., can be observed.
   (d) The simulation can be temporarily suspended, or paused, not only to view information but also to reassign values or redirect entities.
8. Graphical interfaces are recommended for accomplishing verification & validation.

8b) Describe with a neat diagram iterative process of calibrating a model. Which are three steps that aid in the validation process? 10 M
Calibration and Validation of Models

- Verification and validation although are conceptually distinct, usually are conducted simultaneously by the modeler.

- Validation is the overall process of comparing the model and its behavior to the real system and its behavior.

- Calibration is the iterative process of comparing the model to the real system, making adjustments to the model, comparing again and so on.

- The following figure 7.2 shows the relationship of the model calibration to the overall validation process.

- The comparison of the model to reality is carried out by variety of test.

- Tests are subjective and objective.
- Subjective test usually involve people, who are knowledgeable about one or more aspects of the system, making judgments about the model and its output.
- Objective tests always require data on the system's behavior plus the corresponding data produced by the model.

As an aid in the validation process, Naylor and Finger [1967] formulated a three step approach which has been widely followed:-

1. Build a model that has high face validity.
2. Validate model assumptions.
3. Compare the model input-output transformations to corresponding input-output transformations for the real system.

June 2010

1a) What is simulation? Explain with flow chart, the steps involved in simulation study 10 M
Simulation
A Simulation is the imitation of the operation of a real-world process or system over time.

Brief Explanation
- The behavior of a system as it evolves over time is studied by developing a simulation model.
- This model takes the form of a set of assumptions concerning the operation of the system.
The simulation model building can be broken into 4 phases.

**I Phase**
- Consists of steps 1 and 2
- It is period of discovery/orientation
- The analyst may have to restart the process if it is not fine-tuned
- Recalibrations and clarifications may occur in this phase or another phase.

**II Phase**
- Consists of steps 3, 4, 5, 6 and 7
- A continuing interplay is required among the steps
- Exclusion of model user results in implications during implementation

**III Phase**
- Consists of steps 8, 9 and 10
- Conceives a thorough plan for experimenting
- Discrete-event stochastic is a statistical experiment
- The output variables are estimates that contain random error and therefore proper statistical analysis is required.

**IV Phase**
- Consists of steps 11 and 12
- Successful implementation depends on the involvement of user and every steps successful completion.

1b) Differentiate between continuous and discrete systems.  5 M

Ans: i) discrete system

Systems in which the changes are predominantly discontinuous are called discrete systems. Ex: Bank – the number of customer’s changes only when a customer arrives or when the service provided a customer is completed.

![Graph showing number of customers waiting in line over time]

ii) continuous system

Systems in which the changes are predominantly smooth are called continuous system. Ex: Head of a water behind a dam.
1c) What is system and system environment? List the components of a system, with example. 5 M

**Systems**
A system is defined as an aggregation or assemblage of objects joined in some regular interaction or interdependence toward the accomplishment of some purpose.

**Components of a System**

**Entity**: An entity is an object of interest in a system.
Ex: In the factory system, departments, orders, parts and products are the entities.

**Attribute**: An attribute denotes the property of an entity.
Ex: Quantities for each order, type of part, or number of machines in a Department are attributes of factory system.

**Activity**: Any process causing changes in a system is called as an activity.
Ex: Manufacturing process of the department.

**State of the System**: The state of a system is defined as the collection of variables necessary to describe a system at any time, relative to the objective of study. In other words, state of the system mean a description of all the entities, attributes and activities as they exist at one point in time.

**Event**: An event is defined as an instantaneous occurrence that may change the state of the system.

**System Environment**: The external components which interact with the system and produce necessary changes are said to constitute the system environment. In modeling systems, it is necessary to decide on the boundary between the system and its environment. This decision may depend on the purpose of the study.
Ex: In a factory system, the factors controlling arrival of orders may be considered to be outside the factory but yet a part of the system environment. When, we consider the demand and supply of goods, there is certainly a relationship between the factory output and arrival of orders. This relationship is considered as an activity of the system.

June 2010

2a) A grocery store has one checkout counter. Customers arrive at this checkout counter at random from 1 to 8 minutes apart and each interval time has the same probability of occurrence. The service times vary from 1 to 6 minutes with probability given below:

<table>
<thead>
<tr>
<th>Services (minutes)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>probability</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.25</td>
<td>0.10</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Simulate the arrival of 6 customers and calculate:

- Average waiting time for a customer
- Probability that a customer has to wait
- Probability of a server being idle
- Average service time
- Use time between arrival
- Use the following sequence of random numbers:

```
Random digit for arrival: 913 727 015 948 309 922
Random digit for service time: 84 10 74 53 17 79
```

Assume that the first customer arrives at time 0. Depict the simulation in a tabular form. 10 M.
1. The average waiting time for a customer is 2.8 minutes. This is determined in the following manner:

\[
\text{Average Waiting Time} = \frac{\text{Total time customers wait in queue (min)}}{\text{Total no. of customers}}
\]

\[
= \frac{56}{20} = 2.8 \text{ minutes}
\]

2. The probability that a customer has to wait in the queue is 0.65. This is determined in the following manner:

\[
\text{Probability (wait)} = \frac{\text{number of customers who wait}}{\text{Total number of customers}}
\]

\[
= \frac{13}{20} = 0.65
\]

3. The fraction of idle time of the server is 0.21. This is determined in the following manner:

\[
\text{Probability of idle server} = \frac{\text{Total idle time of server (minutes)}}{\text{Total run time of simulation (minutes)}}
\]

\[
= \frac{18}{86} = 0.21
\]

The probability of the server being busy is the complement of 0.21, or 0.79.

4. The average service time is 3.4 minutes, determined as follows:

\[
\text{Average service time (minutes)} = \frac{\text{Total service time}}{\text{Total number of customers}}
\]

\[
= \frac{68}{20} = 3.4 \text{ minutes}
\]
This result can be compared with the expected service time by finding the mean of the service-time distribution using the equation

\[ E(s) = \Sigma sp(s) \]

Applying the expected-value equation to the distribution in table 2.7 gives an expected service time of:

\[ = 1(0.10) + 2(0.20) + 3(0.30) + 4(0.25) + 5(0.10) + 6(0.50) \]
\[ = 3.2 \text{ minutes} \]

The expected service time is slightly lower than the average time in the simulation. The longer simulation, the closer the average will be to \( E(S) \).

5. The average time between arrivals is 4.3 minutes. This is determined in the following manner:

\[ \text{Average time between arrivals (minutes)} = \frac{\text{Sum of all times between arrivals (minutes)}}{\text{Number of arrivals} - 1} \]
\[ = \frac{82}{19} = 4.3 \text{ minutes} \]

2b) Briefly define any four concepts used in discrete event simulation. 10 M

The major concepts are briefly defined and then illustrated with examples:

- **System**: A collection of entities (e.g., people and machines) that interact together over time to accomplish one or more goals.

- **Model**: An abstract representation of a system, usually containing structural, logical, or mathematical relationships which describe a system in terms of state, entities and their attributes, sets, processes, events, activities, and delays.

- **System state**: A collection of variables that contain all the information necessary to describe the system at any time.

- **Entity**: Any object or component in the system which requires explicit representation in the model (e.g., a server, a customer, a machine).

- **Attributes**: The properties of a given entity (e.g., the priority of a customer, the routing of a job through a job shop).

2c) Explain event scheduling algorithm by generating system snapshots at clock=t and clock=t_i

**The Event-Scheduling/Time-Advance Algorithm**
The mechanism for advancing simulation time and guaranteeing that all events occur in correct chronological order is based on the future event list (FEL).

This list contains all event notices for events that have been scheduled to occur at a future time.

At any given time t, the FEL contains all previously scheduled future events and their associated event times.

The FEL is ordered by event time, meaning that the events are arranged chronologically; that is, the event times satisfy

\[ t < t_1 \leq t_2 \leq t_3 \leq \ldots \leq t_n \]

\( t \) is the value of CLOCK, the current value of simulated time. The event dated with time \( t_1 \) is called the imminent event; that is, it is the next event will occur. After the system snapshot at simulation time \( \text{CLOCK} \equiv t \) has been updated, the CLOCK is advanced to simulation time \( \text{CLOCK} = t_1 \) and the imminent event notice is removed from the FEL and the event executed. This process repeats until the simulation is over.

The sequence of actions which a simulator must perform to advance the clock system snapshot is called the event-scheduling/time-advance algorithm.

### Old system snapshot at time \( t \)

<table>
<thead>
<tr>
<th>( \text{CLOCK} )</th>
<th>( \text{System State} )</th>
<th>( \text{Future Event List} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( (5,1,6) )</td>
<td>( (3, t_1) ) — Type 3 event to occur at time ( t_1 ) \n (1, t_2) — Type 1 event to occur at time ( t_2 ) \n (1, t_3) — Type 1 event to occur at time ( t_3 ) \n (2, t_n) — Type 2 event to occur at time ( t_n )</td>
</tr>
</tbody>
</table>

**Step 2.** Advance \( \text{CLOCK} \) to imminent event time (i.e., advance \( \text{CLOCK} \) from \( r \) to \( t_1 \)).

**Step 3.** Execute imminent event: update system state, change entity attributes, and set membership as needed.

**Step 4.** Generate future events (if necessary) and place their event notices on PEL ranked by event time. (Example: Event 4 to occur at time \( t^* \), where \( t_2 < t^* < t_3 \).)

**Step 5.** Update cumulative statistics and counters.

---

3a) Six dump trucks are used to haul coal from the entrance of a small mine to railroad. Each truck is loaded by one of two loaders. After loading truck moves to scale, to be weighed as soon
as possible. Both the loader and the scale have first come first served waiting line for
trucks. Travel time from a loader to scale is considered negligible. After weighing a truck begins
to travel time and then returns to loader queue. The activities of loading, weighing and travel
time are given in the following table.

<table>
<thead>
<tr>
<th>Loading time</th>
<th>10</th>
<th>5</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>10</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighing time</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>16</td>
<td>16</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Travel time</td>
<td>60</td>
<td>100</td>
<td>40</td>
<td>40</td>
<td>80</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

End of simulation is completion of two weighings from the scale. Depict the simulation table and
estimate the loader and scale utilizations. Assume that five of the trucks are at the loaders and
one is at the scale at time 0.

**Average Loader Utilization** = \( \frac{44}{2} = 0.92 \)

**Average Scale Utilization** = \( \frac{24}{24} = 1.00 \)

3b) Define a discrete random variable. Explain the binomial distribution. 5 M
Discrete Random Variables

- **X** is a discrete random variable if the number of possible values of **X** is finite, or countably infinite.
- **Example:** Consider jobs arriving at a job shop.
  - Let **X** be the number of jobs arriving each week at a job shop.
  - **R** = possible values of **X** \( \{0, 1, 2, \ldots \} \)
  - \( p(x) = \text{probability the random variable is } x = P(X = x) \)

\( p(x), i = 1, 2, \ldots \) must satisfy:

1. \( p(x_i) \geq 0 \), for all \( i \)
2. \( \sum_{i} p(x_i) = 1 \)

The collection of pairs \([x_i, p(x_i)], i = 1, 2, \ldots,\) is called the probability distribution of **X**, and \( p(x_i) \) is called the probability mass function (pmf) of **X**.

**Binomial Distribution**

- The number of successes in \( n \) Bernoulli trials, **X**, has a binomial distribution.
  - \( p(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \ldots, n \)

  The number of outcomes having the required number of successes and failures
  Probability that there are \( x \) successes and \((n-x)\) failures

\( \sum_{x} p(x) = 1 \)

- The mean, \( E(x) = p + p + \ldots + p = n*p \)
- The variance, \( V(X) = pq + pq + \ldots + pq = n*pq \)

**4a) Explain the characteristics of a queuing system. List different queuing notations. 10 M**

1.1 Characteristics of Queuing Systems

Key elements of queuing systems:
- **Customer:** refers to anything that arrives at a facility and requires service, e.g., people, machines, trucks, emails.
- **Server:** refers to any resource that provides the requested service, e.g., repairpersons, retrieval machines, runways at airport.
- **Calling population**: the population of potential customers, may be assumed to be finite or infinite.
  - **Finite population model**: if arrival rate depends on the number of customers being served and waiting, e.g., model of one corporate jet, if it is being repaired, the repair arrival rate becomes zero.
  - **Infinite population model**: if arrival rate is not affected by the number of customers being served and waiting, e.g., systems with large population of potential customers.

- **System Capacity**: a limit on the number of customers that may be in the waiting line or system.
  - **Limited capacity**, e.g., an automatic car wash only has room for 10 cars to wait in line to enter the mechanism.
  - **Unlimited capacity**, e.g., concert ticket sales with no limit on the number of people allowed to wait to purchase tickets.

- **For infinite-population models**:
  - In terms of interarrival times of successive customers.
  - Random arrivals: interarrival times usually characterized by a probability distribution.
    - Most important model: Poisson arrival process (with rate $l$), where $A_n$ represents the interarrival time between customer $n-1$ and customer $n$, and is exponentially distributed (with mean $1/l$).
  - Scheduled arrivals: interarrival times can be constant or constant plus or minus a small random amount to represent early or late arrivals.
    - e.g., patients to a physician or scheduled airline flight arrivals to an airport.
  - At least one customer is assumed to always be present, so the server is never idle, e.g., sufficient raw material for a machine.

- **For finite-population models**:
  - Customer is pending when the customer is outside the queueing system, e.g., machine-repair problem: a machine is “pending” when it is operating, it becomes “not pending” the instant it demands service form the repairman.
  - Runtime of a customer is the length of time from departure from the queueing system until that customer’s next arrival to the queue, e.g., machine-repair problem, machines are customers and a runtime is time to failure.
  - Let $A_1^{(i)}, A_2^{(i)}, \ldots$ be the successive runtimes of customer $i$, and $S_1^{(i)}, S_2^{(i)}$ be the corresponding successive system times:

```
Machine 3: Pending    Open (system time)  Pending    Open (system time)
  A_1^{(3)}  S_1^{(3)}  A_2^{(3)}  S_2^{(3)}
First arrival of machine 3  Second arrival of machine 3
```

- **Queue behavior**: the actions of customers while in a queue waiting for service to begin, for example:
  - **Balk**: leave when they see that the line is too long,
  - **Renege**: leave after being in the line when its moving too slowly,
  - **Jockey**: move from one line to a shorter line.

- **Queue discipline**: the logical ordering of customers in a queue that determines which customer is chosen for service when a server becomes free, for example:
First-in-first-out (FIFO)

Last-in-first-out (LIFO)

Service in random order (SIRO)

Shortest processing time first (SPT)

Service according to priority (PR).

- Service times of successive arrivals are denoted by $S_1$, $S_2$, $S_3$.
  - May be constant or random.
  - $\{S_1, S_2, S_3, \ldots\}$ is usually characterized as a sequence of independent and identically distributed random variables, e.g., exponential, Weibull, gamma, lognormal, and truncated normal distribution.

- A queueing system consists of a number of service centers and interconnected queues.
  - Each service center consists of some number of servers, $c$, working in parallel, upon getting to the head of the line, a customer takes the $1^{st}$ available server.

4b) A tool crib has exponential interarrival and service times, and it serves a very large group of mechanics. The mean time between arrivals is 4 minutes. It takes 3 minutes on the average for a tool crib attendant to service a mechanic. The attendant is paid $10 per hour and the mechanic is paid $15 per hour. Would it be advisable to have a second tool crib attendant? 10 M

- Let $T_i$ denote the total time during $[0,T]$ in which the system contained exactly $i$ customers, the time-weighted-average number in a system is defined by:

\[
\hat{L} = \frac{1}{T} \sum_{i=0}^{\infty} iT_i = \sum_{i=0}^{\infty} \left( \frac{T_i}{T} \right)
\]

- The time-weighted-average number in queue is:

\[
\hat{L}_Q = \frac{1}{T} \sum_{i=0}^{\infty} iT_i^Q = \frac{1}{T} \int_0^T L_Q(t) dt \rightarrow L_Q \quad \text{as} \quad T \rightarrow \infty
\]

\[
\hat{L} = \frac{[0(3)+1(12)+2(4)+3(1)]}{20} = 23/20 = 1.15 \text{ customers}
\]

5a) Using multiplicative congruential method, generate random numbers to complete cycle. Explain maximum density and maximum period, $A=11, M=16, X_0=7$ 10 M

The sequence of $X_i$ and subsequent $R_i$ values is computed as follows:

$X_0 = 27$

$X_1 = (17.27 + 43) \mod 100 = 502 \mod 100 = 2$

$R_1 = 2/100 = 0.02$

$X_2 = (17 \cdot 2 + 43) \mod 100 = 77 \mod 100 = 77$

$R_2 = 77/100 = 0.77$

*3pt* $X_3 = (17 \cdot 77 + 43) \mod 100 = 1352 \mod 100 = 52$

$R_3 = 52/100 = 0.52$
First, notice that the numbers generated from Equation (7.2) can only assume values from the set \( I = \{0, 1/m, 2/m, \ldots, (m-1)/m\} \), since each \( X_i \) is an integer in the set \( \{0,1,2,\ldots, m - 1\} \). Thus, each \( R_i \) is discrete on \( I \), instead of continuous on the interval \([0, 1]\). This approximation appears to be of little consequence, provided that the modulus \( m \) is a very large integer. (Values such as \( m = 231 - 1 \) and \( m = 248 \) are in common use in generators appearing in many simulation languages.) By maximum density is meant that the values assumed by \( R_i = 1, 2, \ldots \), leave no large gaps on \([0,1]\).

Second, to help achieve maximum density, and to avoid cycling (i.e., recurrence of the same sequence of generated numbers) in practical applications, the generator should have the largest possible period. Maximal period can be achieved by the proper choice of \( a, c, m, \) and \( X_0 \).

- For \( m \) a power of 2, say \( m = 2^b \) and \( c \neq 0 \), the longest possible period is \( P = m = 2^b \), which is achieved provided that \( c \) is relatively prime to \( m \) (that is, the greatest common factor of \( c \) and \( m \) is 1), and \( a = 1 + 4k \), where \( k \) is an integer.

- For \( m \) a power of 2, say \( m = 2^b \) and \( c = 0 \), the longest possible period is \( P = m/4 = 2^{b-2} \), which is achieved provided that the seed \( X_0 \) is odd and the multiplier \( a \), is given by \( a = 3 + 8K \), for some \( K = 0, 1, \ldots \).

- For \( m \) a prime number and \( c = 0 \), the longest possible period is \( P = m-1 \), which is achieved provided that the multiplier \( a \), has the property that the smallest integer \( k \) such that \( a^k \) is divisible by \( m \) is \( k = m-1 \).

5b) Using suitable frequency test find out whether the random numbers generated are uniformly distributed on the interval \([0,1]\) can be rejected. Assume \( \alpha = 0.05 \) and \( D_{0.05} = 0.565 \). The random numbers are 0.54, 0.73, 0.98, 0.11, 0.68.

Ans: First, the numbers must be ranked from smallest to largest. The calculations can be facilitated by use of Table 7.2. The top row lists the numbers from smallest \( R(1) \) to largest \( R(n) \). The computations for \( D^+ \), namely \( i/N - R(i) \) and for \( D^- \), namely \( R(i) - (i - 1) / N \), are easily accomplished using Table 7.2. The statistics are computed as \( D^+ = 0.26 \) and \( D^- = 0.21 \). Therefore, \( D = \max\{0.26, 0.21\} = 0.26 \). The critical value of \( D \), obtained from Table A.8 for \( a = 0.05 \) and \( N = 5 \), is 0.565. Since the computed value, 0.26, is less than the tabulated critical value, 0.565, the hypothesis of no difference between the distribution of the generated numbers and the uniform distribution is not rejected.

<table>
<thead>
<tr>
<th>Table 7.2. Calculations for Kolmogorov-Smirnov Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{(i)} )</td>
</tr>
<tr>
<td>( i/N )</td>
</tr>
<tr>
<td>( i/N - R_{(i)} )</td>
</tr>
<tr>
<td>( R_{(i)} - (i-1)/N )</td>
</tr>
</tbody>
</table>

The calculations in Table 7.2 are illustrated in Figure 7.2, where the empirical cdf, \( SN(X) \), is compared to the uniform cdf, \( F(x) \). It can be seen that \( D^+ \) is the largest deviation of \( SN(x) \) above \( F(x) \).
and that \( D^+ \) is the largest deviation of \( SN(X) \) below \( F(x) \). For example, at \( R(3) \) the value of \( D^+ \) is given by \( \frac{3}{5} - R(3) = 0.60 - 0.44 = 0.16 \) and of \( D^- \) is given by \( R(3) = \frac{2}{5} = 0.44 - 0.40 = 0.04 \). Although the test statistic \( D \) is defined by Equation (7.3) as the maximum deviation over all \( x \), it can be seen from Figure 7.2 that the maximum deviation will always occur at one of the jump points \( R(1), R(2), \ldots \), and thus the deviation at other values of \( x \) need not be considered.

6a) Suggest a step by step procedure to generate random variates using inverse transform technique for exponential distribution. 6 M

**Inverse Transform Technique:**
The inverse transform technique can be used to sample from exponential, the uniform, the Weibull, and the triangular distributions and empirical distributions. Additionally, it is the underlying principle for sampling from a wide variety of discrete distributions. The technique will be explained in detail for the exponential distribution and then applied to other distributions. It is the most straightforward, but always the most efficient, technique computationally.

**EXPONENTIAL DISTRIBUTION:**

The exponential distribution, has probability density function (pdf) given by

\[
f(X) = \lambda e^{-\lambda x}, \quad x \geq 0
\]

\[
0, \quad x < 0
\]

and cumulative distribution function (cdf) given by

\[
f(X) = \int_{-\infty}^{x} f(t) \, dt = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}
\]

The parameter can be interpreted as the mean number of occurrences per time unit.

For example, if interarrival times \( X_1, X_2, X_3, \ldots \) had an exponential distribution with rate \( \lambda \), then could be interpreted as the mean number of arrivals per time unit, or the arrival rate. Notice that for any \( j \)

\[
E(X_i) = 1/\lambda
\]

so that is the mean interarrival time. The goal here is to develop a procedure for generating values \( X_1, X_2, X_3, \ldots \) which have an exponential distribution.

The inverse transform technique can be utilized, at least in principle, for any distribution. But it is most useful when the cdf, \( F(x) \), is of such simple form that its inverse, \( F^{-1} \), can be easily computed. A step-by-step procedure for the inverse transform technique illustrated by me exponential distribution, is as follows:

Step 1. Compute the cdf of the desired random variable \( X \). For the exponential distribution, the cdf is

\[
F(x) = 1 - e^{-\lambda x}, \quad x > 0.
\]

Step 2. Set \( F(X) = R \) on the range of \( X \). For the exponential distribution, it becomes \( 1 - e^{-\lambda x} = R \) on the range \( x > 0 \). Since \( X \) is a random variable (with the exponential distribution in this case), it follows that \( 1 - R \) is also a random variable, here called \( R \). As will be shown later, \( R \) has a uniform distribution over the interval \((0,1)\).
Step 3. Solve the equation $F(X) = R$ for $X$ in terms of $R$. For the exponential distribution, the solution proceeds as follows:

\[
1 - e^{-\lambda x} = R \\
e^{-\lambda x} = 1 - R \\
-\lambda x = \ln(1 - R) \\
x = -\frac{1}{\lambda} \ln(1 - R) \\
\text{(5.1)}
\]

Equation (5.1) is called a random-variate generator for the exponential distribution. In general, Equation (5.1) is written as $X = F^{-1}(R)$. Generating a sequence of values is accomplished through steps 4.

Step 4. Generate (as needed) uniform random numbers $R_1, R_2, R_3,...$ and compute the desired random variates by

\[
X_i = F^{-1}(R_i)
\]

For the exponential case, $F(R) = (-1/\lambda)\ln(1 - R)$ by Equation (5.1), so that

\[
X_i = -\frac{1}{\lambda} \ln (1 - R_i) \quad (5.2)
\]

for $i = 1, 2, 3,...$. One simplification that is usually employed in Equation (5.2) is to replace $1 - R_i$ by $R_i$ to yield

\[
X_i = -\frac{1}{\lambda} \ln R_i \quad (5.3)
\]

which is justified since both $R_i$ and $1 - R_i$ are uniformly distributed on $(0,1)$.

6b) Enlist the steps involved in development of a useful model of input data. 6 M

There are four steps in the development of a useful model of input data:

- Collect data from the real system of interest. This often requires a substantial time and resource commitment. Unfortunately, in some situations it is not possible to collect data.

- Identify a probability distribution to represent the input process. When data are available, this step typically begins by developing a frequency distribution, or histogram, of the data.

- Choose parameters that determine a specific instance of the distribution family. When data are available, these parameters may be estimated from the data.

- Evaluate the chosen distribution and the associated parameters for good-of-fit. Goodness-of-fit may be evaluated informally via graphical methods, or formally via statistical tests. The chi-square and the Kolmo-gorov-Smirnov tests are standard goodness-of-fit tests. If not satisfied that the chosen distribution is a good approximation of the data, then the analyst returns to the second step, chooses a different family of distributions, and repeats the procedure. If several iterations of this procedure fail to yield a fit between an assumed distributional form and the collected data.

7a) Briefly explain the measures of performance of a simulation system. 10 M

Measures of performance
Consider the estimation of a performance parameter, \( q \) (or \( f \)), of a simulated system.

- Discrete time data: \([Y_1, Y_2, \ldots, Y_n]\), with ordinary mean: \( q \)

- Continuous-time data: \( \{Y(t), 0 \leq t \leq T_E\} \) with time-weighted mean: \( f \)

**Point Estimation for Discrete Time Data**

- The point estimator:
  \[
  \hat{\theta} = \frac{1}{n} \sum_{i=1}^{n} Y_i
  \]

  Is unbiased if its expected value is \( \theta \), that is if:
  \[
  E(\hat{\theta}) = \theta
  \]

**Point Estimator**

**Point Estimation for Continuous-Time Data**

- The point estimator:
  \[
  \hat{\phi} = \frac{1}{T_E} \int_0^{T_E} Y(t) dt
  \]

  - Is biased in general where:
  - An unbiased or low-bias estimator is desired.

- Usually, system performance measures can be put into the common framework of \( q \) or \( f \):
  e.g., the proportion of days on which sales are lost through an out-of-stock situation, let:
  \[
  Y(t) = \begin{cases} 
  1, & \text{if out of stock on day } i \\
  0, & \text{otherwise}
  \end{cases}
  \]

- Performance measure that does not fit: quantile or percentile:
  - Estimating quantiles: the inverse of the problem of estimating a proportion or probability.
    \[
    \Pr\{Y \leq \theta\} = p
    \]
  - Consider a histogram of the observed values \( Y \):

Find such that 100\( p \)% of the histogram is to the left of (smaller than)

7b) **Explain the distinction between terminating or transient simulation and steady state simulation. Give examples.**

- A terminating simulation: runs over a simulated time interval \([0, T_E]\).
  
  A common goal is to estimate:
  \[
  \theta = E\left( \frac{1}{n} \sum_{i=1}^{n} Y_i \right), \quad \text{for discrete output}
  \]

  \[
  \phi = E\left( \frac{1}{T_E} \int_0^{T_E} Y(t) dt \right), \quad \text{for continuous output } Y(t), 0 \leq t \leq T_E
  \]
In general, independent replications are used, each run using a different random number stream and independently chosen initial conditions.

8a) Explain with a neat diagram model building, verification and validation process  10 M

Verification of Simulation Models

- The purpose of model verification is to assure that the conceptual model is reflected accurately in the computerized representation.

- The conceptual model quite often involves some degree of abstraction about system operations, or some amount of simplification of actual operations.

Many suggestions can be given for use in the verification process:

1: Have the computerized representation checked by someone other than its developer.
2: Make a flow diagram which includes each logically possible action a system can take when an event occurs, and follow the model logic for each action for each event type.
3: Closely examine the model output for reasonableness under a variety of settings of Input parameters.
4. Have the computerized representation print the input parameters at the end of the Simulation to be sure that these parameter values have not been changed inadvertently.
5. Make the computerized representation of self-documenting as possible.
6. If the computerized representation is animated, verify that what is seen in the animation imitates the actual system.
7. The interactive run controller (IRC) or debugger is an essential component of Successful simulation model building. Even the best of simulation analysts makes mistakes or commits logical errors when building a model. The IRC assists in finding and correcting those errors in the follow ways:
   (a) The simulation can be monitored as it progresses.
   (b) Attention can be focused on a particular line of logic or multiple lines of logic that constitute a procedure or a particular entity.
(c) Values of selected model components can be observed. When the simulation has paused, the current value or status of variables, attributes, queues, resources, counters, etc., can be observed.
(d) The simulation can be temporarily suspended, or paused, not only to view information but also to reassign values or redirect entities.
8. Graphical interfaces are recommended for accomplishing verification & validation.

8b) Describe the three steps approach to validation by Naylor and Finger. 10 M

Calibration and Validation of Models
• Verification and validation although are conceptually distinct, usually are conducted simultaneously by the modeler.
• Validation is the overall process of comparing the model and its behavior to the real system and its behavior.
• Calibration is the iterative process of comparing the model to the real system, making adjustments to the model, comparing again and so on.
• The following figure 7.2 shows the relationship of the model calibration to the overall validation process.
• The comparison of the model to reality is carried out by variety of test.
• Tests are subjective and objective.
• Subjective test usually involve people, who are knowledgeable about one or more aspects of the system, making judgments about the model and its output.
• Objective tests always require data on the system's behavior plus the corresponding data produced by the model.

As an aid in the validation process, Naylor and Finger [1967] formulated a three step approach which has been widely followed:

1. Build a model that has high face validity.
2. Validate model assumptions.
3. Compare the model input-output transformations to corresponding input-output transformations for the real system.

Dec -2011

1a) List any five circumstances, When the Simulation is appropriate tool and when it is not?

**Ans: When Simulation is the Appropriate Tool**

- Simulation enables the study of and experimentation with the internal interactions of a complex system, or of a subsystem within a complex system.
- Informational, organizational and environmental changes can be simulated and the effect of those alternations on the model’s behavior can be observed.
- The knowledge gained in designing a simulation model can be of great value toward suggesting improvement in the system under investigation.
- By changing simulation inputs and observing the resulting outputs, valuable insight may be obtained into which variables are most important and how variables interact.
- Simulation can be used as a pedagogical device to reinforce analytic solution methodologies. Simulation can be used to experiment with new designs or policies prior to implementation, so as to prepare for what may happen.
- Simulation can be used to verify analytic solutions.
- By simulating different capabilities for a machine, requirements can be determined.

**When Simulation is Not Appropriate**

- Simulation should be used when the problem cannot be solved using common sense.
- Simulation should not be used if the problem can be solved analytically.
- Simulation should not be used, if it is easier to perform direct experiments.
- Simulation should not be used, if the costs exceeds savings.
- Simulation should not be performed, if the resources or time are not available.

1b) Explain the steps in simulation study. With flow chart?10M

**Steps in a Simulation study**

1. **Problem formulation**
   Every study begins with a statement of the problem, provided by policy makers. Analyst ensures its clearly understood. If it is developed by analyst policy makers should understand and agree with it.

2. **Setting of objectives and overall project plan**
   The objectives indicate the questions to be answered by simulation. At this point a determination should be made concerning whether simulation is the appropriate methodology. Assuming it is appropriate, the overall project plan should include
   - A statement of the alternative systems
   - A method for evaluating the effectiveness of these alternatives
   - Plans for the study in terms of the number of people involved Cost of the study
   - The number of days required to accomplish each phase of the work with the anticipated results.

3. **Model conceptualization**
   The construction of a model of a system is probably as much art as science. The art of modeling is enhanced by an ability
   - To abstract the essential features of a problem
To select and modify basic assumptions that characterize the system
To enrich and elaborate the model until a useful approximation results
Thus, it is best to start with a simple model and build toward greater complexity. Model conceptualization enhance the quality of the resulting model and increase the confidence of the model user in the application of the model.

4. Data collection
There is a constant interplay between the construction of model and the collection of needed input data. Done in the early stages. Objective kind of data are to be collected.

5. Model translation
Real-world systems result in models that require a great deal of information storage and computation. It can be programmed by using simulation languages or special purpose simulation software. Simulation languages are powerful and flexible. Simulation software models development time can be reduced.

6. Verified
It pertains to the computer program and checking the performance. If the input parameters and logical structure and correctly represented, verification is completed.

7. Validated
It is the determination that a model is an accurate representation of the real system. Achieved through calibration of the model, an iterative process of comparing the model to actual system behavior and the discrepancies between the two.

8. Experimental Design
The alternatives that are to be simulated must be determined. Which alternatives to simulate may be a function of runs. For each system design, decisions need to be made concerning

- Length of the initialization period
- Length of simulation runs
- Number of replication to be made of each run

9. Production runs and analysis
They are used to estimate measures of performance for the system designs that are being simulated.

10. More runs
Based on the analysis of runs that have been completed. The analyst determines if additional runs are needed and what design those additional experiments should follow.

11. Documentation and reporting
Two types of documentation.

- Program documentation
- Process documentation

Program documentation
Can be used again by the same or different analysts to understand how the program operates. Further modification will be easier. Model users can change the input parameters for better performance.

Process documentation
Gives the history of a simulation project. The result of all analysis should be reported clearly and concisely in a final report. This enable to review the final formulation and alternatives, results of the experiments and the recommended solution to the problem. The final report provides a vehicle of certification.

12. Implementation
Success depends on the previous steps. If the model user has been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.

2a) Six dump trucks are used to have coal from the entrance of a mine to a railroad. Each truck is loaded by one by one of the four loaders. After loading, a truck immediately moves to the scale, to be weighed as soon as possible. Both the loader and the scale have first-come first-served Waiting line for trucks. Travel time from a loader to scale is considered negligible. After Being Weighed, a truck begins travel time [during which time truck unloads] and then afterwards returns to loader queue. The activities of loading, Weighing and travel time are given in the following table:

End of simulation is completion of Four Weighings from the scale. Depict the simulation table estimate the loader and scale utilizations. Assume that Five of the trucks are at the loaders and one is at the scale at Time θ.
<table>
<thead>
<tr>
<th>Clock t</th>
<th>LQ(t)</th>
<th>L(t)</th>
<th>WQ(t)</th>
<th>W(t)</th>
<th>Loader Queue</th>
<th>Weigh Queue</th>
<th>FEL</th>
<th>B_s</th>
<th>B_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td></td>
<td>DT4</td>
<td>DT3, DT6</td>
<td>(EL, 5, DT3) (EL, 10, DT2) (EW, 12, DT1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td>DT3</td>
<td>DT6</td>
<td>(EL, 10, DT2) (EL, 5 + 5, DT4) (EW, 12, DT1)</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
<td>DT6</td>
<td>DT3, DT2</td>
<td>(EL, 10, DT4) (EW, 12, DT1) (EL, 10 + 10, DT5)</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td></td>
<td>DT3</td>
<td>DT4</td>
<td>(EL, 20, DT1) (EL, 10 + 15, DT6)</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Clock t</th>
<th>LQ(t)</th>
<th>L(t)</th>
<th>WQ(t)</th>
<th>W(t)</th>
<th>Loader Queue</th>
<th>Weigh Queue</th>
<th>FEL</th>
<th>B_s</th>
<th>B_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>DT2, DT4</td>
<td></td>
<td>(EL, 20, DT3) (EW, 12 + 12, DT3) (EL, 12 + 60, DT1)</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>DT2, DT4, DT5</td>
<td></td>
<td>(EL, 24, DT3) (EL, 25, DT6) (ALQ, 72, DT1)</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>DT4, DT5</td>
<td></td>
<td>(EL, 25, DT6) (EW, 24 + 12, DT2) (ALQ, 72, DT1) (ALQ, 24 + 100, DT3)</td>
<td>44</td>
<td>24</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>DT4, DT5, DT6</td>
<td></td>
<td>(EW, 36, DT2) (ALQ, 72, DT1) (ALQ, 124, DT3)</td>
<td>45</td>
<td>25</td>
</tr>
</tbody>
</table>
2B) Explain simulation in GPSS With a block diagram, for the single server queue simulation?

2c) Explain the following.
   System: A collection of entities that interact together over time to accomplish one or more goals.
   Event List: A list of event notices for future events, ordered by time of occurrence; known as the future event list (FEL).
   □ Always ranked by the event time
   Entity: An object in the system that requires explicit representation in the model, e.g., people, machines, nodes, packets, server, customer
   Event: An instantaneous occurrence that changes the state of a system.

3a) Explain discrete random variable and continuous random variable with an example?

Discrete Random Variables

- X is a discrete random variable if the number of possible values of X is finite, or countably infinite.
- Example: Consider jobs arriving at a job shop.
  - Let X be the number of jobs arriving each week at a job shop.
  - \( R_x = \) possible values of X (range space of X) = \( \{0, 1, 2, \ldots\} \)
  - \( p(x) = \) probability the random variable is \( x_i = P(X = x_i) \)
  □ \( p(x), i = 1, 2, \ldots \) must satisfy:
    1. \( p(x_i) \geq 0 \), for all \( i \)
    2. \( \sum_{i=1}^{\infty} p(x_i) = 1 \)
  □ The collection of pairs \( [x_i, p(x_i)], i = 1, 2, \ldots, \) is called the probability distribution of X, and \( p(x) \) is called the probability mass function (pmf) of X.
Continuous Random Variables [Probability Review]

- Example: Life of an inspection device is given by $X$, a continuous random variable with pdf:

\[
f(x) = \begin{cases} 
    \frac{1}{2} e^{-x/2}, & x \geq 0 \\
    0, & \text{otherwise}
\end{cases}
\]

- $X$ has an exponential distribution with mean 2 years
- Probability that the device’s life is between 2 and 3 years is:

\[
P(2 \leq x \leq 3) = \frac{1}{2} \int_{2}^{3} e^{-x/2} dx = 0.14
\]

3b) Explain any two discrete distributions? 5 M

Bernoulli Trials and Bernoulli Distribution [Discrete Dist’n]

- Bernoulli Trials:
  - Consider an experiment consisting of $n$ trials, each can be a success or a failure.
    - Let $X_j = 1$ if the $j$th experiment is a success
    - and $X_j = 0$ if the $j$th experiment is a failure
  - The Bernoulli distribution (one trial):

\[
p_j(x_j) = p(x_j) = \begin{cases} 
    p, & x_j = 1, j = 1,2,\ldots,n \\
    1-p = q, & x_j = 0, j = 1,2,\ldots,n \\
    0, & \text{otherwise}
\end{cases}
\]

- where $E(X_j) = p$ and $V(X_j) = p(1-p) = pq$

- Bernoulli process:
  - The $n$ Bernoulli trials where trials are independent:

\[
p(x_1,x_2,\ldots,x_n) = p_1(x_1)p_2(x_2) \ldots p_n(x_n)
\]
3c) explain the following continuous distributions?

i) Uniform Distribution

A random variable $X$ is uniformly distributed on the interval $(a,b)$, $U(a,b)$, if its pdf and cdf are:

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & x \geq b \end{cases}$$

Properties

- $P(x_1 < X < x_2)$ is proportional to the length of the interval $[F(x_2) - F(x_1)] = (x_2 - x_1)/(b-a)$
- $E(X) = (a+b)/2$
- $V(X) = (b-a)^2/12$
- $U(0,1)$ provides the means to generate random numbers, from which random variates can be generated.

ii) Exponential Distribution
Exponential Distribution [Continuous Dist’n]

- A random variable $X$ is exponentially distributed with parameter $\lambda > 0$ if its pdf and cdf are:

$$
    f(x) = \begin{cases} 
        \lambda e^{-\lambda x}, & x \geq 0 \\
        0, & \text{elsewhere} 
    \end{cases} 
    \quad \text{and} \quad 
    F(x) = \begin{cases} 
        0, & \text{for } x < 0 \\
        \int_0^x \lambda e^{-\lambda t} \, dt = 1 - e^{-\lambda x}, & x \geq 0 
    \end{cases}
$$

- $E(X) = \frac{1}{\lambda}$
- $V(X) = \frac{1}{\lambda^2}$

- Used to model interarrival times when arrivals are completely random, and to model service times that are highly variable
- For several different exponential pdf’s (see figure), the value of intercept on the vertical axis is $\lambda$, and all pdf’s eventually intersect.

4a) Explain the characteristics of queuing system? And Explain the notations of queuing system? 10 M

**Calling Population** [Characteristics of Queueing System]

- Calling population: the population of potential customers, may be assumed to be finite or infinite.
  - Finite population model: if arrival rate depends on the number of customers being served and waiting, e.g., model of one corporate jet, if it is being repaired, the repair arrival rate becomes zero.
  - Infinite population model: if arrival rate is not affected by the number of customers being served and waiting, e.g., systems with large population of potential customers.

**System Capacity** [Characteristics of Queueing System]

- System Capacity: a limit on the number of customers that may be in the waiting line or system.
  - Limited capacity, e.g., an automatic car wash only has room for 10 cars to wait in line to enter the mechanism.
  - Unlimited capacity, e.g., concert ticket sales with no limit on the number of people allowed to wait to purchase tickets.

**Arrival Process** [Characteristics of Queueing System]

- For infinite-population models:
  - In terms of interarrival times of successive customers.
  - Random arrivals: interarrival times usually characterized by a probability distribution.
Most important model: Poisson arrival process (with rate $l$), where $An$ represents the interarrival time between customer $n-1$ and customer $n$, and is exponentially distributed (with mean $l/\lambda$).

- Scheduled arrivals: interarrival times can be constant or constant plus or minus a small random amount to represent early or late arrivals.
  - e.g., patients to a physician or scheduled airline flight arrivals to an airport.
- At least one customer is assumed to always be present, so the server is never idle, e.g., sufficient raw material for a machine.

Arrival Process [Characteristics of Queueing System]

- For finite-population models:
  - Customer is pending when the customer is outside the queueing system, e.g., machine-repair problem: a machine is “pending” when it is operating, it becomes “not pending” the instant it demands service form the repairman.
  - Runtime of a customer is the length of time from departure from the queueing system until that customer’s next arrival to the queue, e.g., machine-repair problem, machines are customers and a runtime is time to failure.
  - Let $A_1(i), A_2(i), \ldots$ be the successive runtimes of customer $i$, and $S_1(i), S_2(i)$ be the corresponding successive system times:

```
+-----------------+        +-----------------+        +-----------------+
| Machine 3:      |        | Machine 3:      |        | Machine 3:      |
| Pending         |        | Pending         |        | Pending         |
| Open (system time) |        | Open (system time) |        | Open (system time) |
| $A_1^{(3)}$     |        | $S_1^{(3)}$     |        | $A_2^{(3)}$     |
| First arrival of machine 3 |        | Second arrival of machine 3 |        | $S_2^{(3)}$     |
```

Queue Behavior and Queue Discipline [Characteristics of Queueing System]

- Queue behavior: the actions of customers while in a queue waiting for service to begin, for example:
  - Balk: leave when they see that the line is too long,
  - Renege: leave after being in the line when its moving too slowly,
  - Jockey: move from one line to a shorter line.

- Queue discipline: the logical ordering of customers in a queue that determines which customer is chosen for service when a server becomes free, for example:
  - First-in-first-out (FIFO)
  - Last-in-first-out (LIFO)
Service in random order (SIRO)
Shortest processing time first (SPT)
Service according to priority (PR).

Queueing Notation

[Characteristics of Queueing System]

- A notation system for parallel server queues: \( A/B/c/N/K \)
  - \( A \) represents the interarrival-time distribution,
  - \( B \) represents the service-time distribution,
  - \( c \) represents the number of parallel servers,
  - \( N \) represents the system capacity,
  - \( K \) represents the size of the calling population.

Primary performance measures of queueing systems:

- \( P_n \): steady-state probability of having \( n \) customers in system,
- \( P_n(t) \): probability of \( n \) customers in system at time \( t \),
- \( l \): arrival rate,
- \( le \): effective arrival rate,
- \( m \): service rate of one server,
- \( r \): server utilization,
- \( An \): interarrival time between customers \( n-1 \) and \( n \),
- \( Sn \): service time of the \( n \)th arriving customer,
- \( Wn \): total time spent in system by the \( n \)th arriving customer,
- \( WnQ \): total time spent in the waiting line by customer \( n \),
- \( L(t) \): the number of customers in system at time \( t \),
- \( LQ(t) \): the number of customers in queue at time \( t \),
- \( L \): long-run time-average number of customers in system,
- \( LQ \): long-run time-average number of customers in queue,
- \( w \): long-run average time spent in system per customer,
- \( wQ \): long-run average time spent in queue per customer.

4b) Explain any two long run measure of performance queuing system? 10 M
5a) Explain the two different techniques for generating random numbers with examples?

### 1 Linear Congruential Method

Let $T_i$ denote the total time during $[0, T]$ in which the system contained exactly $i$ customers, the time-weighted-average number in a system is defined by:

$$L = \frac{1}{T} \sum T_i - \sum \left( \frac{T_i}{T} \right)$$

Consider the total area under the function is $L(t)$, then,

$$L = \frac{1}{T} \sum T_i = \frac{1}{T} \int_0^T L(t) dt$$

The long-run time-average # in system, with probability 1:

$$\hat{L} = \frac{1}{\infty} \int_0^T L(t) dt \rightarrow L \text{ as } T \rightarrow \infty$$

#### Server Utilization

For $G/G/1/\infty/\infty$ queues:

- Any single-server queueing system with average arrival rate $\lambda$ customers per time unit, where average service time $E(S) = 1/\mu$ time units, infinite queue capacity and calling population.
- Conservation equation, $L = \lambda w$, can be applied.
- For a stable system, the average arrival rate to the server, $\lambda_s$, must be identical to $\lambda$.
- The average number of customers in the server is:

$$\hat{L}_s = \frac{1}{T} \int_0^T (L(t) - L_Q(t)) dt = \frac{T - T_0}{T}$$

#### Server Utilization and System Performance

Example: A physician who schedules patients every 10 minutes and spends $S_i$ minutes with the $i^{th}$ patient:

- Arrivals are deterministic, $A_1 = A_2 = \ldots = \lambda^{-1} = 10$.
- Services are stochastic, $E(S) = 9.3$ min and $V(S) = 0.81$ min$^2$.
- On average, the physician’s utilization $= \rho = \lambda/\mu = 0.93 < 1$.
- Consider the system is simulated with service times: $S_1 = 9$, $S_2 = 12$, $S_3 = 9$, $S_4 = 9$, $S_5 = 9$, $\ldots$. The system becomes:

The occurrence of a relatively long service time ($S_2 = 12$) causes a waiting line to form temporarily.
The linear congruential method, initially proposed by Lehmer [1951], produces a sequence of integers, \( X, X_2, \ldots \) between zero and \( m - 1 \) according to the following recursive relationship:

\[
X_{i+1} = (aX_i + c) \mod m, \quad i = 0, 1, 2, \ldots
\]

The initial value \( X_0 \) is called the seed.

If \( c \neq 0 \) in Equation (7.1), the form is called the mixed congruential method. When \( c = 0 \), the form is known as the multiplicative congruential method. The selection of the values for \( a, c, m \) and \( X_0 \) drastically affects the statistical properties and the cycle length. The random integers are being generated \( [0, m - 1] \), and to convert the integers to random numbers:

2 Combined Linear Congruential Generators

As computing power has increased, the complexity of the systems that we are able to simulate has also increased.

One fruitful approach is to combine two or more multiplicative congruential generators in such a way that the combined generator has good statistical properties and a longer period. The following result from L'Ecuyer [1988] suggests how this can be done: If \( W_i, 1, W_i, 2, \ldots, W_i,k \) are any independent, discrete-valued random variables (not necessarily identically distributed), but one of them, say \( W_i, 1 \), is uniformly distributed on the integers 0 to \( m_i - 2 \), then is uniformly distributed on the integers 0 to \( m_i - 2 \).

\[
W_i = \left( \sum_{j=1}^{k} W_{i,j} \right) \mod m_i - 1
\]

5b) The sequence of numbers 0.44, 0.81, 0.14, 0.05, 0.93 were generated, use the Kolmogorov-Smirnov test with a level of significance \( a \) of 0.05. Compare \( F(X) \) and \( S_n(X) \) 10 M
The calculations in Table 7.2 are illustrated in Figure 7.2, where the empirical cdf, \( SN(X) \), compared to the uniform cdf, \( F(x) \). It can be seen that \( D^+ \) is the largest deviation of \( SN(x) \) above \( F(x) \), and that \( D^- \) is the largest deviation of \( SN(X) \) below \( F(x) \). For example, at \( R(3) \) the value of \( D^+ \) is given by \( 3/5 - R(3) = 0.60 - 0.44 = 0.16 \) and of \( D^- \) is given by \( R(3) = 2/5 = 0.40 - 0.44 = 0.04 \). Although the test statistic \( D \) is defined by Equation (7.3) as the maximum deviation over all \( x \), it can be seen from Figure 7.2 that the maximum deviation will always occur at one of the jump points \( R(1), R(2), \ldots \), and thus the deviation at other values of \( x \) need not be considered.

6a) Explain inverse transform technique of producing random variates for Exponential Distribution? 5 M
Exponential Distribution

- Exponential Distribution:
  - Exponential cdf:
    \[ r = F(x) = 1 - e^{-\lambda x} \quad \text{for } x \geq 0 \]
    \[ R = F(X) \]
    \[ X = F^{-1}(R) \]

- To generate \(X_1, X_2, X_3 \ldots\)

\[
X_i = F^{-1}(R_i) \\
= -(1/\lambda) \ln(1-R_i) \\
= - (1/\lambda) \ln(R_i)
\]

Since both \(1-R_i\) & \(R_i\) are uniformly distributed between 0 & 1

### Generation of Exponential Variates X, with Mean 1, Given Random Numbers

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ri</td>
<td>0.1306</td>
<td>0.0422</td>
<td>0.6597</td>
<td>0.7965</td>
<td>0.7696</td>
<td></td>
</tr>
</tbody>
</table>

Figure: Inverse-transform technique for \(\exp(\lambda = 1)\)

---

6b) Generate Three poison variates with mean \(\alpha=0.2\)  

5M
6c) Explain the types of simulation with respect to the output Analysis? Give at least two examples?

10 M

Type of Simulations

- Terminating verses non-terminating simulations
- Terminating simulation:
  - Runs for some duration of time $T_E$, where $E$ is a specified event that stops the simulation.
  - Starts at time $0$ under well-specified initial conditions.
  - Ends at the stopping time $T_E$.
  - Bank example: Opens at 8:30 am (time $0$) with no customers present and 8 of the 11 teller working (initial conditions), and closes at 4:30 pm (Time $T_E = 480$ minutes).
  - The simulation analyst chooses to consider it a terminating system because the object of interest is one day's operation.
Type of Simulations

- Non-terminating simulation:
  - Runs continuously, or at least over a very long period of time.
  - Examples: assembly lines that shut down infrequently, telephone systems, hospital emergency rooms.
  - Initial conditions defined by the analyst.
  - Runs for some analyst-specified period of time $T_E$.
  - Study the steady-state (long-run) properties of the system, properties that are not influenced by the initial conditions of the model.

- Whether a simulation is considered to be terminating or non-terminating depends on both
  - The objectives of the simulation study and
  - The nature of the system.

7a) Explain the chi-square test with $\alpha = 0.05$ to test whether the data shown below are uniformly distributed. Table 7.3 contains the essential computations. The test uses $n = 10$ intervals of equal length, namely $(0, 0.1], (0.1, 0.2], \ldots, (0.9, 1.0]$. The value of $\chi^2_0$ is 3.4. This is compared with the critical value $\chi^2_{0.05, 9} = 16.9$. Since $\chi^2_0$ is much smaller than the tabulated value of $\chi^2_{0.05, 9}$ the null hypothesis of a uniform distribution is not rejected.

<table>
<thead>
<tr>
<th>Interval</th>
<th>$O_i$</th>
<th>$E_i$</th>
<th>$(O_i - E_i)^2$</th>
<th>$(O_i - E_i)^2 / E_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34</td>
<td>0.90</td>
<td>0.25</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>0.83</td>
<td>0.76</td>
<td>0.79</td>
<td>0.64</td>
<td>0.70</td>
</tr>
<tr>
<td>0.96</td>
<td>0.99</td>
<td>0.77</td>
<td>0.67</td>
<td>0.56</td>
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<tr>
<td>0.47</td>
<td>0.30</td>
<td>0.17</td>
<td>0.82</td>
<td>0.56</td>
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<tr>
<td>0.79</td>
<td>0.71</td>
<td>0.23</td>
<td>0.92</td>
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<td>0.06</td>
<td>0.39</td>
<td>0.84</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>0.18</td>
<td>0.26</td>
<td>0.97</td>
<td>0.44</td>
<td>0.60</td>
</tr>
</tbody>
</table>
7b) Explain chi-square of goodness of fit – test for exponential distribution? 5 M

Compare histogram

Valid for **large** sample sizes

Arrange the **n** observations which **approximately** follows the chi-square distribution with **k-s-1** degrees of freedom, where s = # of parameters of the hypothesized distribution estimated by the sample statistics.

- Null hypothesis – observations come from a specified distribution cannot be rejected at a significance of **α** if:

- **Comments:**
  - Errors in cells with small **E**'s affect the test statistics more than cells with large **E**'s.
  - Minimum size of **E** debated: recommends a value of 5 or more; if not combine adjacent cells.
  - Test designed for discrete distributions and large sample sizes only. For continuous distributions, Chi-Square test is only an approximation (i.e., level of significance holds only for **n-∞**).

Example 1: 500 random numbers generated using a random number generator; observations categorized into cells at intervals of 0.1, between 0 and 1. At level of significance of 0.1, are these numbers IID U(0,1)?

<table>
<thead>
<tr>
<th>Interval</th>
<th>Oi</th>
<th>Ei</th>
<th>(Oi-Ei)^2/Ei</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>10</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>10</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>10</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>10</td>
<td>-2</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

100 100 0 3.4
\[ \chi^2_0 = 5.85; \text{ from the table } \chi^2_{(0.9, 9)} = 14.68; \]

Hypothesis accepted at significance level of 0.10.

8a) Explain a model building, and verification and validation? 10M
Verification

- Purpose:
- Many common-sense suggestions, for example:
  - Have someone else check the model.
  - Make a flow diagram that includes each logically possible action a system can take when an event occurs.
  - Closely examine the model output for reasonableness under a variety of input parameter settings. (Often overlooked!)
  - Print the input parameters at the end of the simulation, make sure they have not been changed inadvertently.

8b) Explain any two output analysis for steady-state simulation? 10M

Initialization Bias [Steady-State Simulations]

- No widely accepted, objective and proven technique to guide how much data to delete to reduce initialization bias to a negligible level.
- Plots can, at times, be misleading but they are still recommended.
  - Ensemble averages reveal a smoother and more precise trend as the # of replications, R, increases.
  - Ensemble averages can be smoothed further by plotting a moving average.
  - Cumulative average becomes less variable as more data are averaged.
  - The more correlation present, the longer it takes for \( \bar{Y} \) to approach steady state.
  - Different performance measures could approach steady state at different rates.
Error Estimation

[Steady-State Simulations]

- For a covariance stationary time series, \( \{Y_1, \ldots, Y_n\} \):
  - Lag-k autocovariance is: \( \hat{\alpha}_k = \text{cov}(Y_i, Y_{i+k}) = \text{cov}(Y_{i+k}, Y_i) \)
  - Lag-k autocorrelation is: \( \rho_k = \frac{\hat{\alpha}_k}{\sigma^2} \)

- If a time series is covariance stationary, then the variance of \( Y \) is:
  \[
  \text{V}(Y) = \frac{\sigma^2}{n} \left[ 1 + 2 \sum_{k=1}^{n-1} \left( 1 - \frac{k}{n} \right) \rho_k \right]
  \]

- The expected value of the variance estimator is:
  \[
  E\left( \frac{S^2}{n} \right) = BV(F), \quad \text{where} \quad B = \frac{n/\epsilon - 1}{n-1}
  \]

Error Estimation

[Steady-State Simulations]

a) Stationary time series \( Y_i \)
   exhibiting positive autocorrelation.

b) Stationary time series \( Y_i \)
   exhibiting negative autocorrelation.

c) Nonstationary time series with
   an upward trend
1) a) What is system And System Environment? Explain the components of a system with an example?

Ans: System

A system is defined as an aggregation or assemblage of objects joined in some regular interaction or interdependence toward the accomplishment of some purpose.
Example: Production System
Production Control System

System Environment

The external components which interact with the system and produce necessary changes are said to constitute the system environment. In modeling systems, it is necessary to decide on the boundary between the system and its environment. This decision may depend on the purpose of the study.

Components of a System

Entity - An entity is an object of interest in a system.
Ex: In the factory system, departments, orders, parts and products are The entities.
Attribute - An attribute denotes the property of an entity.
Ex: Quantities for each order, type of part, or number of machines in a Department are attributes of factory system.
Activity - Any process causing changes in a system is called as an activity.
Ex: Manufacturing process of the department.
State of the System - The state of a system is defined as the collection of variables necessary to describe a system at any time, relative to the objective of study. In other words, state of the system mean a description of all the entities, attributes and activities as they exist at one point in time.
Event - An event is define as an instantaneous occurrence that may change the state of the system.

1b) Explain various steps in simulation study. With help of neat diagram?

1. Problem formulation
Every study begins with a statement of the problem, provided by policy makers. Analyst ensures its clearly understood. If it is developed by analyst policy makers should understand and agree with it.

2. Setting of objectives and overall project plan
The objectives indicate the questions to be answered by simulation. At this point a determination should be made concerning whether simulation is the appropriate methodology. Assuming it is appropriate, the overall project plan should include:
- A statement of the alternative systems
- A method for evaluating the effectiveness of these alternatives
- Plans for the study in terms of the number of people involved
- Cost of the study
- The number of days required to accomplish each phase of the work with the anticipated results.

3. Model conceptualization
The construction of a model of a system is probably as much art as science. The art of modeling is enhanced by an ability:
- To abstract the essential features of a problem
- To select and modify basic assumptions that characterize the system
- To enrich and elaborate the model until a useful approximation results

Thus, it is best to start with a simple model and build toward greater complexity. Model conceptualization enhance the quality of the resulting model and increase the confidence of the model user in the application of the model.

4. Data collection
There is a constant interplay between the construction of model and the collection of needed input data. Done in the early stages. Objective kind of data are to be collected.

5. Model translation
Real-world systems result in models that require a great deal of information storage and computation. It can be programmed by using simulation languages or special purpose simulation software.

6. Verified
It pertains to the computer program and checking the performance. If the input parameters and logical structure and correctly represented, verification is completed.

7. Validated
It is the determination that a model is an accurate representation of the real system. Achieved through calibration of the model, an iterative process of comparing the model to actual system behavior and the discrepancies between the two.

8. Experimental Design
The alternatives that are to be simulated must be determined. Which alternatives to simulate may be a function of runs. For each system design, decisions need to be made concerning:
- Length of the initialization period
- Length of simulation runs
- Number of replication to be made of each run

9. Production runs and analysis
They are used to estimate measures of performance for the system designs that are being simulated.

10. More runs
Based on the analysis of runs that have been completed. The analyst determines if additional runs are needed and what design those additional experiments should follow.

11. Documentation and reporting

Two types of documentation.
- Program documentation
- Process documentation

Program documentation

Can be used again by the same or different analysts to understand how the program operates. Further modification will be easier. Model users can change the input parameters for better performance.

Process documentation

Gives the history of a simulation project. The result of all analysis should be reported clearly and concisely in a final report. This enable to review the final formulation and alternatives, results of the experiments and the recommended solution to the problem. The final report provides a vehicle of certification.

12. Implementation

Success depends on the previous steps. If the model user has been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.

---

2a) With the help of flow diagram, Explain the single channel queuing system. 10M

Ans: Simulation of Queuing systems
A queueing system is described by its calling population, the nature of the arrivals, the service mechanism, the system capacity, and the queueing discipline. A single-channel queueing system is portrayed in figure 1.

- In the single-channel queue, the calling population is infinite; that is, if a unit leaves the calling population and joins the waiting line or enters service, there is no change in the arrival rate of other units that may need service.
- Arrivals for service occur one at a time in a random fashion; once they join the waiting line, they are eventually served. In addition, service times are of some random length according to a probability distribution which does not change over time.
- The system capacity has no limit, meaning that any number of units can wait in line.
- Finally, units are served in the order of their arrival by a single server or channel.
- Arrivals and services are defined by the distributions of the time between arrivals and the distribution of service times, respectively.
- For any simple single or multichannel queue, the overall effective arrival rate must be less than the total service rate, or the waiting line will grow without bound. When queues grow without bound, they are termed “explosive” or unstable.
- The state of the system is the number of units in the system and the status of the server, busy or idle.
- An event is a set of circumstances that cause an instantaneous change in the state of the system. In a single–channel queueing system there are only two possible events that can affect the state of the system.
  - They are the entry of a unit into the system
  - The completion of service on a unit.
The queueing system includes the server, the unit being serviced, and units in the queue. The simulation clock is used to track simulated time. If a unit has just completed service, the simulation proceeds in the manner shown in the flow diagram of figure 2. Note that the server has only two possible states: it is either busy or idle.

The arrival event occurs when a unit enters the system. The flow diagram for the arrival event is shown in figure 3. The unit may find the server either idle or busy; therefore, either the unit begins service immediately, or it enters the queue for the server. The unit follows the course of action shown in fig 4.

![Flow Diagram](image)

**Figure 2: Service-just-completed flow diagram**

If the server is busy, the unit enters the queue. If the server is idle and the queue is empty, the unit begins service. It is not possible for the server to be idle and the queue to be nonempty.

2b) A large milling machine has three different bearings that fail in service. The cumulative distribution function of the life of each bearing is identical given in table 1. When a bearing fails, the mill stops, a repair person is called and a new bearing is installed. The delay time of the repair person’s arriving at the milling machine is also a random variable, with the distribution given in table 2. Downtime for the mill is estimated at $5/minute. The direct on site cost of the repair person is $15/hour. It takes 20 minutes to change 1 bearing, 30 minutes to change 2 bearings and 40 minutes to change 3 bearings. The bearing cost is $16 each. A proposal has been made to replace all 3 bearings whenever a bearing fails. Management needs an evaluation of this
propose. Simulate the system for 10,000 hours of operation under proposed method and determine the total cost of the proposed system.

Table 1 Bearing life distribution

<table>
<thead>
<tr>
<th>Bearing Life(Hrs)</th>
<th>1000</th>
<th>1100</th>
<th>1200</th>
<th>1300</th>
<th>1400</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
<th>1900</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.10</td>
<td>0.13</td>
<td>0.25</td>
<td>0.13</td>
<td>0.09</td>
<td>0.12</td>
<td>0.02</td>
<td>0.06</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 2 Delay time distribution

<table>
<thead>
<tr>
<th>Delay(minutes)</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Consider the following sequence of random digits for bearing life times

<table>
<thead>
<tr>
<th>Bearing 1</th>
<th>67</th>
<th>8</th>
<th>49</th>
<th>84</th>
<th>44</th>
<th>30</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing 2</td>
<td>70</td>
<td>43</td>
<td>86</td>
<td>93</td>
<td>81</td>
<td>44</td>
<td>19</td>
<td>51</td>
</tr>
<tr>
<td>Bearing 3</td>
<td>76</td>
<td>65</td>
<td>61</td>
<td>96</td>
<td>65</td>
<td>56</td>
<td>11</td>
<td>86</td>
</tr>
</tbody>
</table>

Consider the following sequence of random digits for delay time

<table>
<thead>
<tr>
<th>Delay</th>
<th>3</th>
<th>7</th>
<th>5</th>
<th>1</th>
<th>4</th>
<th>3</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
</table>
2.3 Other Examples of Simulation (2)

- Example 2.5 (Cont.)

The cumulative distribution function of the life of each bearing is identical, as shown in Table 2.22.

The delay time of the repairperson’s arriving at the milling machine is also a random variable, with the distribution given in Table 2.23.

<table>
<thead>
<tr>
<th>Table 2.22 Bearing-Life Distribution</th>
<th>Table 2.23 Delay-Time Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bearing Life (Hours)</strong></td>
<td><strong>Probability</strong></td>
</tr>
<tr>
<td>1,000</td>
<td>0.10</td>
</tr>
<tr>
<td>1200</td>
<td>0.25</td>
</tr>
<tr>
<td>1,400</td>
<td>0.12</td>
</tr>
<tr>
<td>1,600</td>
<td>0.08</td>
</tr>
<tr>
<td>1,800</td>
<td>0.05</td>
</tr>
<tr>
<td>2,000</td>
<td>0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Delay Time (Minutes)</strong></th>
<th><strong>Probability</strong></th>
<th><strong>Cumulative Probability</strong></th>
<th><strong>Random-Digit Assignment</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>15</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

- The cumulative distribution function of the life of each bearing is identical, as shown in Table 2.22.

Table 2.24 Bearing Replacement Using Current Method

<table>
<thead>
<tr>
<th>Bearing 1</th>
<th>Bearing 2</th>
<th>Bearing 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accumulated Life (Hours)</strong></td>
<td><strong>Delay (Minutes)</strong></td>
<td><strong>Accumulated Life (Hours)</strong></td>
</tr>
<tr>
<td><strong>Bearing Number</strong></td>
<td><strong>Bearing Number</strong></td>
<td><strong>Bearing Number</strong></td>
</tr>
<tr>
<td>1</td>
<td>1,400</td>
<td>1,400</td>
</tr>
<tr>
<td>2</td>
<td>1,200</td>
<td>2,400</td>
</tr>
<tr>
<td>3</td>
<td>4,900</td>
<td>1,500</td>
</tr>
<tr>
<td>4</td>
<td>84</td>
<td>2,500</td>
</tr>
<tr>
<td>5</td>
<td>44</td>
<td>1,200</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>1,200</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>1,000</td>
</tr>
<tr>
<td>8</td>
<td>63</td>
<td>1,400</td>
</tr>
<tr>
<td>9</td>
<td>02</td>
<td>1,000</td>
</tr>
<tr>
<td>10</td>
<td>02</td>
<td>2,000</td>
</tr>
<tr>
<td>11</td>
<td>77</td>
<td>1,500</td>
</tr>
<tr>
<td>12</td>
<td>59</td>
<td>1,300</td>
</tr>
<tr>
<td>13</td>
<td>23</td>
<td>1,100</td>
</tr>
<tr>
<td>14</td>
<td>53</td>
<td>1,300</td>
</tr>
<tr>
<td>15</td>
<td>85</td>
<td>1,700</td>
</tr>
<tr>
<td>16</td>
<td>75</td>
<td>1,500</td>
</tr>
</tbody>
</table>

*RD*, random digit.
3a) what do you mean by World View? Discuss the different types of World View? 10M

**Ans: World Views**

- **Process-interaction**
  - Modeler thinks in terms of processes
  - A process is the lifecycle of one entity, which consists of various events and activities
  - Simulation model is defined in terms of entities or objects and their lifecycle as they flow through the system, demanding resources and queuing to wait for resources
  - Some activities might require the use of one or more resources whose capacities are limited
  - Processes interact, e.g., one process has to wait in a queue because the resource it needs is busy with another process
  - A process is a time-sequenced list of events, activities, and delays, including demands for resources that define the lifecycle of one entity as it moves through a system
  - Variable time advance

![Diagram of process-interaction](image)

- **Activity-scanning**
  - Modeler concentrates on activities of a model and those conditions that allow an activity to begin
  - At each clock advance, the conditions for each activity are checked, and if the conditions are true, then the corresponding activity begins
  - **Fixed time advance**
  - Disadvantage: The repeated scanning to discover whether an activity can begin results in slow runtime
  - Improvement: Three-phase approach
    - Combination of events scheduling with activity scanning
Events are activities of duration zero time units

Two types of activities

- **Bactivities**: activities bound to occur all primary events and unconditional activities
- **Cactivities**: activities or events that are conditional upon certain conditions being true

The B-type activities can be scheduled ahead of time, just as in the event-scheduling approach

- Variable time advance
- FEL contains only B-type events

  Scanning to learn whether any C-type activities can begin or C-type events occur happen only at the end of each time advance, after all B-type events have completed.

3b) The maximum inventory level, M, is 11 units and the review period, N, is 5 days. The problem is to estimate, by simulation, the average ending units in inventory and the number of days when a shortage condition occurs. The distribution of the number of units demanded per day is shown in Table 9. In this example, lead-time is a random variable, as shown in Table 10. Assume that orders are placed at the close of business and are received for inventory at the beginning as determined by the lead-time.

ANS: Simulation of an (M, N) Inventory System

**Table 2.19 Random-Digit Assignments for Daily Demand**

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
<th>Cumulative Probability</th>
<th>Random-Digit Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.10</td>
<td>0.10</td>
<td>01–10</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.35</td>
<td>11–35</td>
</tr>
<tr>
<td>2</td>
<td>0.35</td>
<td>0.70</td>
<td>36–70</td>
</tr>
<tr>
<td>3</td>
<td>0.21</td>
<td>0.91</td>
<td>71–91</td>
</tr>
<tr>
<td>4</td>
<td>0.09</td>
<td>1.00</td>
<td>92–00</td>
</tr>
</tbody>
</table>
4 a) What is the role of maximum density and maximum period in generation of random numbers? With given seed 27, constant multiplier 17, increment 43 and modulus 100. And generate Five random numbers?

Ans: These random integers should appear to be uniformly distributed the integers zero to 99.

Random numbers between zero and 1 can be generated by

\[ R_i = \frac{X_i}{m}, \text{ i = 1, 2, ..... (7.2)} \]

The sequence of \( X_i \) and subsequent \( R_i \) values is computed as follows:

\[ X_0 = 27 \]
\[ X_1 = (17 \cdot 27 + 43) \mod 100 = 502 \mod 100 = 2 \quad R_1 = \frac{2}{100} = 0.02 \]
\[ X_2 = (17 \cdot 2 + 43) \mod 100 = 77 \mod 100 = 77 \quad R_2 = \frac{77}{100} = 0.77 \]
\[ X_3 = (17 \cdot 77 + 43) \mod 100 = 1352 \mod 100 = 52 \quad R_3 = \frac{52}{100} = 0.52 \]

4 b) Out of syllabus
5 a) what is inverse transform technique? Derive an expression for Exponential Distribution?

Random-Variate Generation
- Develop understanding of generating samples from a specified distribution as input to a simulation model.
- Illustrate some widely-used techniques for generating random variates.
  - Inverse-transform technique
  - Acceptance-rejection technique
  - Special properties

**Inverse-transform Technique**
- The concept:
  - For cdf function: \( r = F(x) \)
  - Generate \( r \) from uniform \((0,1)\)
  - Find \( x \):
    \[
    x = F^{-1}(r)
    \]

**Exponential Distribution**
- Exponential Distribution:
  - Exponential cdf:
  - To generate \( X_1, X_2, X_3 \ldots \)
    \[
    X_i = F^{-1}(R_i) = -\frac{1}{\lambda} \ln(R_i)
    \]
    Since both \( 1 - R_i \) & \( R_i \) are uniformly distributed between 0& 1

5 b) A Sequence of 1000 four digits has been generated and analysis indicates the following combinations & frequencies, Based on poker test check whether the numbers are independent. Use \( \alpha=0.05 \), \( X^2_{0.05,2}=5.99 \)

<table>
<thead>
<tr>
<th>Combinations (i)</th>
<th>( O_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four different digit</td>
<td>565</td>
</tr>
<tr>
<td>One fair</td>
<td>17</td>
</tr>
<tr>
<td>Two Pair</td>
<td>24</td>
</tr>
<tr>
<td>Three like digit</td>
<td>2</td>
</tr>
</tbody>
</table>
### Ans:

<table>
<thead>
<tr>
<th>Combination i</th>
<th>Observed $O_i$</th>
<th>Expected $E_i$</th>
<th>$(O_i - E_i)^2/E_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 different digit</td>
<td>565</td>
<td>504</td>
<td>7.3 8</td>
</tr>
<tr>
<td>1 pair</td>
<td>392</td>
<td>432</td>
<td>3.7 0</td>
</tr>
<tr>
<td>2 pair</td>
<td>17</td>
<td>27</td>
<td>3.7 0</td>
</tr>
<tr>
<td>3 like digits</td>
<td>24</td>
<td>36</td>
<td>3.2 7</td>
</tr>
<tr>
<td>4 like digits</td>
<td>2</td>
<td>1</td>
<td>3.2 7</td>
</tr>
<tr>
<td>Total</td>
<td>1000</td>
<td>1000</td>
<td>18.060 6</td>
</tr>
</tbody>
</table>

$X^2_{0.05,3}=7.81 < X^2_0=18.0806$

6a) What is Acceptence-Rejection Technique? Generate three poison variates with mean $\alpha=0.2$  

**Ans:** Procedure of generating a Poisson random variate $N$ is as follows

1. Set $n=0$, $P=1$
2. Generate a random number $R_{n+1}$, and replace $P$ by $P \cdot (R_{n+1})$
3. If $P < \exp(-\alpha)$, then accept $N=n$
   - Otherwise, reject the current $n$, increase $n$ by one, and return to step 2.
7a) What do you mean by verification and validation of simulation models? Explain calibration and validation of models with neat diagram? 

6 b) out of syllabus

**Example: Generate three Poisson variates with mean \( \alpha = 0.2 \)**

- \( \exp(-0.2) = 0.8187 \)

**Variate 1**

- Step 1: Set \( n = 0 \), \( P = 1 \)
- Step 2: \( R_1 = 0.4357 \), \( P = 1 \times 0.4357 \)
- Step 3: Since \( P = 0.4357 < \exp(-0.2) \), accept \( N = 0 \)

**Variate 2**

- Step 1: Set \( n = 0 \), \( P = 1 \)
- Step 2: \( R_1 = 0.4146 \), \( P = 1 \times 0.4146 \)
- Step 3: Since \( P = 0.4146 < \exp(-0.2) \), accept \( N = 0 \)

**Variate 3**

- Step 1: Set \( n = 0 \), \( P = 1 \)
- Step 2: \( R_1 = 0.8353 \), \( P = 1 \times 0.8353 \)
- Step 3: Since \( P = 0.8353 > \exp(-0.2) \), reject \( n = 0 \) and return to Step 2 with \( R_1 = 0.9952 \), \( P = 0.8353 \times 0.9952 = 0.8313 \)
- Step 3: Since \( P = 0.8313 > \exp(-0.2) \), reject \( n = 1 \) and return to Step 2 with \( R_2 = 0.8004 \), \( P = 0.8313 \times 0.8004 = 0.6654 \)
- Step 3: Since \( P = 0.6654 < \exp(-0.2) \), accept \( N = 2 \)

**Verification**

- **Purpose:**
- **Many common-sense suggestions, for example:**
  - Have someone else check the model.
  - Make a flow diagram that includes each logically possible action a system can take when an event occurs.
  - Closely examine the model output for reasonableness under a variety of input parameter settings. *(Often overlooked!)*
  - Print the input parameters at the end of the simulation, make sure they have not been changed inadvertently.
Modeling-Building, Verification & Validation

Purpose & Overview

- The goal of the validation process is:
  - To produce a model that represents true behavior closely enough for decision-making purposes
  - To increase the model’s credibility to an acceptable level

- Validation is an integral part of model development
  - Verification – building the model correctly (correctly implemented with good input and structure)
  - Validation – building the correct model (an accurate representation of the real system)

- Most methods are informal subjective comparisons while a few are formal statistical procedures
7b) Discuss types of simulations With respect of output Analysis with Example?  

Type of Simulations

- Terminating verses non-terminating simulations
- Terminating simulation:
  - Runs for some duration of time $T_E$, where $E$ is a specified event that stops the simulation.
  - Starts at time $0$ under well-specified initial conditions.
  - Ends at the stopping time $T_E$.
  - Bank example: Opens at 8:30 am (time $0$) with no customers present and 8 of the 11 teller working (initial conditions), and closes at 4:30 pm (Time $T_E = 480$ minutes).
  - The simulation analyst chooses to consider it a terminating system because the object of interest is one day’s operation.
Type of Simulations

- Non-terminating simulation:
  - Runs continuously, or at least over a very long period of time.
  - Examples: assembly lines that shut down infrequently, telephone systems, hospital emergency rooms.
  - Initial conditions defined by the analyst.
  - Runs for some analyst-specified period of time $T_E$.
  - Study the steady-state (long-run) properties of the system, properties that are not influenced by the initial conditions of the model.

- Whether a simulation is considered to be terminating or non-terminating depends on both
  - The objectives of the simulation study and
  - The nature of the system.

8 Write a Short note on 20 M

a) Characteristics of queuing system?

Characteristics of Queueing Systems

- Key elements of queueing systems:
  - Customer: refers to anything that arrives at a facility and requires service, e.g., people, machines, trucks, emails.
  - Server: refers to any resource that provides the requested service, e.g., repairpersons, retrieval machines, runways at airport.

Calling Population  [Characteristics of Queueing System]

- Calling population: the population of potential customers, may be assumed to be finite or infinite.
- Finite population model: if arrival rate depends on the number of customers being served and waiting, e.g., model of one corporate jet, if it is being repaired, the repair arrival rate becomes zero.

- Infinite population model: if arrival rate is not affected by the number of customers being served and waiting, e.g., systems with large population of potential customers.

System Capacity [Characteristics of Queueing System]

- System Capacity: a limit on the number of customers that may be in the waiting line or system.
  - Limited capacity, e.g., an automatic car wash only has room for 10 cars to wait in line to enter the mechanism.
  - Unlimited capacity, e.g., concert ticket sales with no limit on the number of people allowed to wait to purchase tickets.

Arrival Process [Characteristics of Queueing System]

- For infinite-population models:
  - In terms of interarrival times of successive customers.
  - Random arrivals: interarrival times usually characterized by a probability distribution.
    - Most important model: Poisson arrival process (with rate \( l \)), where \( A_n \) represents the interarrival time between customer \( n-1 \) and customer \( n \), and is exponentially distributed (with mean \( 1/l \)).
  - Scheduled arrivals: interarrival times can be constant or constant plus or minus a small random amount to represent early or late arrivals.
    - e.g., patients to a physician or scheduled airline flight arrivals to an airport.
  - At least one customer is assumed to always be present, so the server is never idle, e.g., sufficient raw material for a machine.

b) Errors while generating pseudorandom numbers

Pseudo means false, so false random numbers are being generated. The goal of any generation scheme, is to produce a sequence of numbers between zero and 1 which simulates, or initiates, the ideal properties of uniform distribution and independence as closely as possible. When
generating pseudo-random numbers, certain problems or errors can occur. These errors, or departures from ideal randomness, are all related to the properties stated previously.

Some examples include the following

1. The generated numbers may not be uniformly distributed.
2. The generated numbers may be discrete-valued instead continuous valued
3. The mean of the generated numbers may be too high or too low.
4. The variance of the generated numbers may be too high or low
5. There may be dependence. The following are examples:
   (a) Autocorrelation between numbers.
   (b) Numbers successively higher or lower than adjacent numbers.

c) Network of Queue

Networks of Queues

- Many systems are naturally modeled as networks of single queues: customers departing from one queue may be routed to another.
- The following results assume a stable system with infinite calling population and no limit on system capacity:
  - Provided that no customers are created or destroyed in the queue, then the departure rate out of a queue is the same as the arrival rate into the queue (over the long run).
  - If customers arrive to queue $i$ at rate $\lambda_i$, and a fraction $0 \leq \rho_{ij} \leq 1$ of them are routed to queue $j$ upon departure, then the arrival rate form queue $i$ to queue $j$ is $\lambda_i \rho_{ij}$ (over the long run).
Networks of Queues

- The overall arrival rate into queue \( j \):
  \[
  \lambda_j = a_j + \sum_{i \neq j} \hat{\lambda}_i p_{ij}
  \]

- If queue \( j \) has \( c_j < \infty \) parallel servers, each working at rate \( \mu_j \), then the long-run utilization of each server is \( \rho_j = \frac{\lambda_j}{c_j \mu_j} \) (where \( \rho_j < 1 \) for stable queue).

- If arrivals from outside the network form a Poisson process with rate \( a_j \) for each queue \( j \), and if there are \( c_j \) identical servers delivering exponentially distributed service times with mean \( 1/\mu_j \), then, in steady state, queue \( j \) behaves like an \( M/M/c_j \) queue with arrival rate \( \lambda_j = a_j + \sum_{i \neq j} \hat{\lambda}_i p_{ij} \).

**d) Hypothesis Testing in Simulation**

**Hypothesis Testing**

[Bank Example: Validate I-O Transformation]

- Conduct the \( t \) test:
  - Chose level of significance (\( \alpha = 0.5 \)) and sample size (\( n = 6 \)), see result in Table 10.2.
  - Compute the same mean and sample standard deviation over the \( n \) replications:
    \[
    \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_{i}, \quad S = \frac{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}{n-1} = 0.81 \text{ minutes}
    \]
  - Compute test statistics:
    \[
    |t| = \frac{\bar{Y}_n - \mu_0}{S/\sqrt{n}} = \frac{2.51 - 4.3}{0.827/\sqrt{6}} = 5.24 > t_{critical} = 2.571 \quad \text{(for a 2-sided test)}
    \]
  - Hence, reject \( H_0 \). Conclude that the model is inadequate.
  - Check: the assumptions justifying a \( t \) test, that the observations \( (Y_{ni}) \) are normally and independently distributed.