PIPE FLOW

INTRODUCTION

A pipe is a closed conduit carrying a fluid under pressure. Fluid motion in a pipe is subjected to a certain resistance. Such a resistance is assumed to be due to Friction. Mainly due to the viscous property of the fluid.
Fluid flow in pipes is of considerable importance in many of the processes, like;

- Animals and Plants circulation systems.
- In our homes.
- City water.
- Irrigation system.
- Sewer water system, etc

To describe any of these flows, conservation of mass and conservation of momentum equations are the most general forms could be used to describe the dynamic system. The key issue is the relation between flow rate and pressure drop.

CLASSIFICATION OF PIPE FLOW

Based on the values of a non-dimensional number known as Reynold's number (Re), the flow in the pipe can classified.
The Reynold's Number (Re) is defined as the ratio of Inertia force of a flowing fluid and the Viscous force. It is mathematically expressed as,
Re=(Inertia force/Viscous force) =(ρ V D/μ)
Where, ρ is mass density
V is average velocity of flow
D is diameter of pipe
μ is dynamic viscosity of flowing fluid

Based on the values of Reynold's number (Re), flow is classified as Follows:
Laminar flow
Turbulent flow
Trasition flow

Laminar Flow or Viscous Flow

In the laminar flow, the stream lines are practically parallel to each other or flow takes place In the form of telescopic tubes.
Occurs when Reynold's number Re< 2000.

Viscous forces are more predominant compared to inertia Forces when the flow velocity or discharge is low, i.e, under laminar flow condition

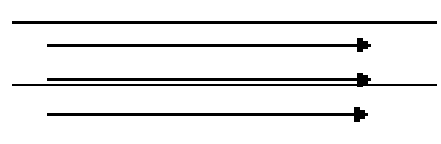
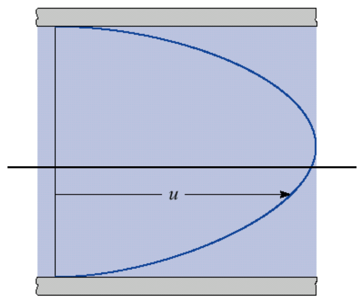


Fig. 1 Laminar Flow

In laminar flow velocity increases gradually from zero at the boundary to Maximum at the center (Fig. 2).
Laminar flow is regular and smooth and velocity at any point practically remains constant in magnitude & direction.
Therefore, the flow is also known as stream Line flow.



(a)

Fig. 2 Velocity Distribution in Laminar Flow

In the laminar flow conditions, there will be no exchange of fluid particles from one layer to another and no momentum transmission from one layer to another.
Ex: Flow of thick oil in narrow tubes, flow of Ground Water, Flow of Blood in blood vessels.

Turbulent Flow

In the turbulent flow the fluid flow at higher flow rates, the streamlines are not steady and straight and the flow is not laminar. Flow is very much disorder and there will be violent mixing as shown in Fig.3. The turbulent flow occurs ***when Reynolds*** number above 4000.
Fluid velocity at a point varies randomly with time. Generally, the flow field will vary in both space and time with fluctuations that comprise "turbulence" (Fig. 4)

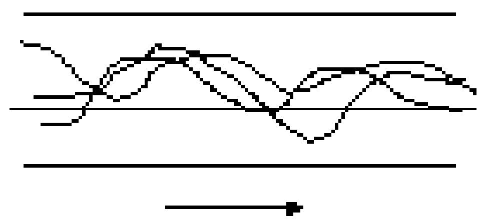


Fig. 3 Turbulent Flow

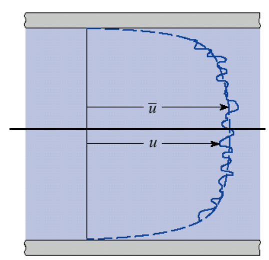


Fig.4 Velocity Distribution in Turbulent Flow

Transitional Flow:

In this, the condition is neither the laminar nor turbulent. It is intermittently turbulent flow. The stream lines get disturbed a little.
This type of flow occurs when 2000< Re < 4000.

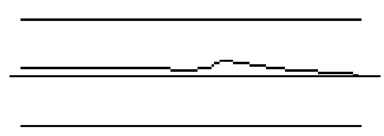


Fig. 3 Transitional Flow

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A good example of laminar and turbulent flow is the rising smoke from a incense stick (agara bathi) or cigarette.
The smoke initially travels in smooth, straight lines (laminar flow) then starts to “wave” back and forth (transition flow) and finally seems to randomly mix (turbulent flow).

Fig. 4 Smoke from Incense Stick

Losses in Pipe Flow

Losses in pipe flow can be two types viz:-

a)Major Loss
b)Minor Loss

a)Major Loss: As the name itself indicates, this is the largest of the losses in a pipe. This loss occurs due to friction only. Hence, it is known as head loss due to friction (h_f)

b)Minor Loss: Minor losses in a pipe occurs due to change in magnitude or direction of flow.

Minor losses are classified as (i) Entry Loss, (ii) Exit loss, (iii) Sudden expansion loss (iv) Sudden contraction loss (v) Losses due to bends & pipe fittings.

These losses must be calculated so that, for example:

- the proper size and number of pumps can be specified in the design of a municipal water distribution system;
- the conduit size for a gravity-flow urban drainage project may be determined;
- the optimum size of valves and the radius of curvature of elbows can be stipulated in the specifications of a pipeline design.

When the ratio of the length of the pipeline, , to the diameter, , exceeds 2000:1, pipe system energy losses are predominantly the result of pipe friction.

The energy losses resulting from pipe appurtenances are termed "minor" losses and are usually neglected in the calculation of pipe system energy losses.
In short lengths of pipe, however, these minor losses can become major sources of energy loss.

Energy loss is usually called power loss or head loss.
Head loss is the measure of the reduction in the total head (sum of elevation head, velocity head and pressure head) of the fluid as it moves through a fluid system.
Head loss includes friction loss and local loss minor losses)

The Darcy-Weisbach equation is used to express energy loss caused by pipe friction

Head Loss due to Friction

Consider the flow through a straight horizontal pipe of diameter D, Length L, between two sections (1) & (2) as shown. Let P1 & P2 be the pressures at these sections. τ_0 is the shear stress acting along the pipe boundary.

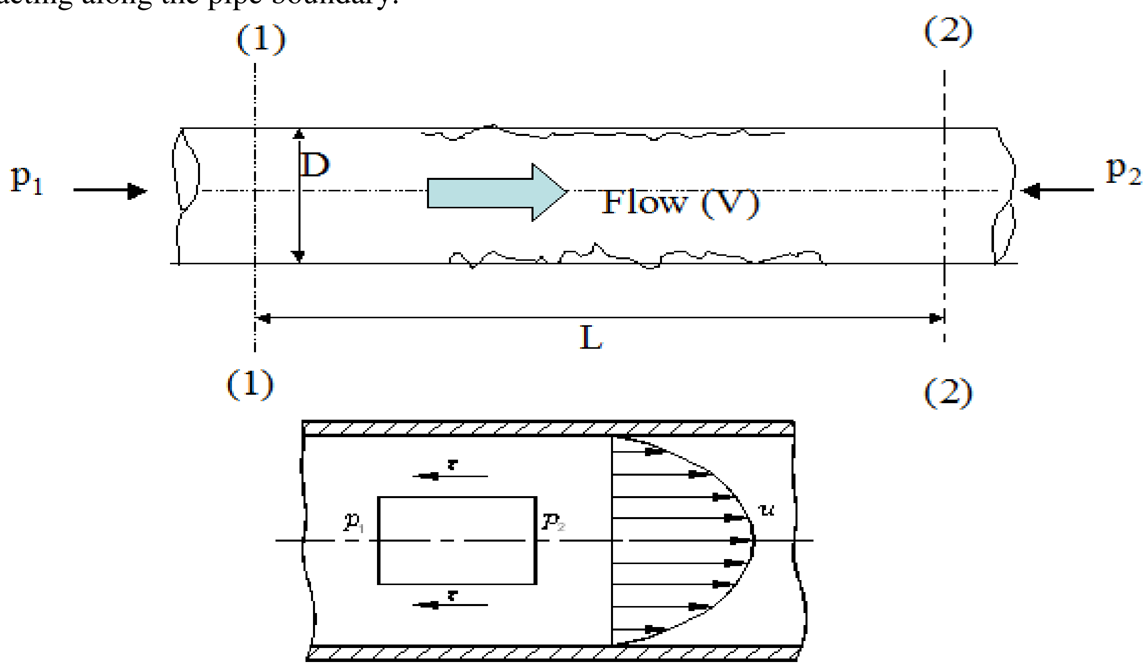


Fig. 5 Various Components of Head Loss through pipe

From II Law of Newton
Force = Mass x accn. But acceleration = 0, as there is no change in velocity, the reason that pipe diameter is uniform or same throughout.

$$\therefore \Sigma forces = 0$$

$$i.e. \quad +P_1\frac{\pi D^2}{4} - P_2\frac{\pi D^2}{4} - \tau_0 x \pi D L = 0$$

$$(P_1 - P_2)\frac{\pi D^2}{4} = \tau_0 \pi D L$$

$$or (P_1 - P_2) = \frac{4\tau_0 L}{D} --- (1)$$

Applying Bernoulli's equation between (1) & (2) with the centre line of the pipe as datum & considering head loss due to friction hf,.

$$Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_f$$

$Z_1 = Z_2 \because$ Pipe is horizontal

$V_1 = V_2 \because$ Pipe diameter is same throughout

$$\therefore \frac{P_1 - P_2}{\gamma} = h_f \;---(2)$$

Substituting eq (2) in eq.(1)

$$h_f x \gamma = \frac{4\tau_0 L}{D}$$

$$or \quad \tau_0 = \frac{h_f \gamma D}{4L} \;---(3)$$

From Experiments, Darcy Found that

$$\tau_0 = \frac{f}{8}\rho V^2 \;---(4)$$

f=Darcy's friction factor (property of the pipe materials Mass density of the liquid.
V = average velocity
Substituting eq (4) in eq.(3)

$$\frac{f}{8}\rho V^2 = \frac{h_f \gamma D}{4L} \quad \text{or} \quad h_f = \frac{4Lf\rho V^2}{8\gamma D}$$

But

$$\frac{\gamma}{\rho} = g$$

$$\therefore h_f = \left(\frac{fLV^2}{2gD}\right) ---(5)$$

From Continuity equation, Q= AxV, V= Q/A

$$V = \frac{4Q}{\pi D^2}$$

Substituting for V in Eq. 5,

$$\therefore h_f = \left(\frac{8fLQ^2}{gh^2D^5}\right) ---(6)$$

Equations (5) & (6) are known as DARCY – WEISBACH Equation

Pipes in Series or Compound Pipe

The compound pipe (or pipe n series) is an arrangement made by connecting different diameters of pipe with a common axis as shown in figure.

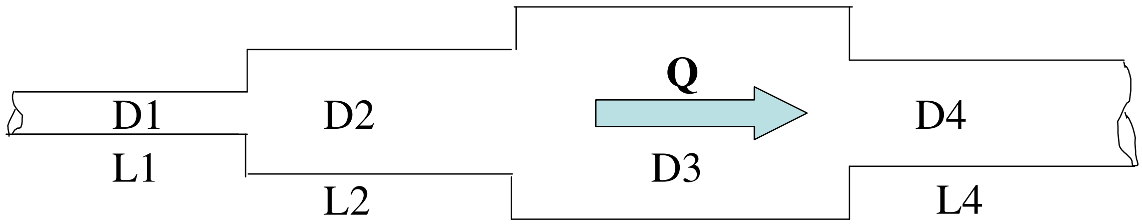


Fig. 6 Compound Pipe OR Pipes in Series

Let D1, D2, D3, D4 be the diameters of the pies as shown in figure..
Let L1, L2,L3, L4 are lengths of a number of Pipes connected in series
The (hf)1, (hf)2, (hf)3 & (hf)4 are the head loss due to friction for each pipe.

The total head loss due to friction, h_f , for the entire pipe system is the summation of each of the head loss occurring in all he pipes, which is given by,

$$h_f = hf_1 + hf_2 + hf_3 + hf_4$$

I.e.,

$$h_f = \frac{f L_1 V_1^2}{2g D_1} + \frac{f L_2 V_2^2}{2g D_2} + \frac{f L_3 V_3^2}{2g D_3} + \frac{f L_4 V_4^2}{2g D_4}$$

Or

$$h_f = \frac{8 f L_1 Q^2}{g \pi^2 D_1^5} + \frac{8 f L_2 Q^2}{g \pi^2 D_2^5} + \frac{8 f L_3 Q^2}{g \pi^2 D_3^5} + \frac{8 f L_4 Q^2}{g \pi^2 D_4^5}$$

PIPES IN PARALLEL

The below figure shows the arrangement of pipes in parallel. As it can be seen from the figure, the pipes are parallel to each other.

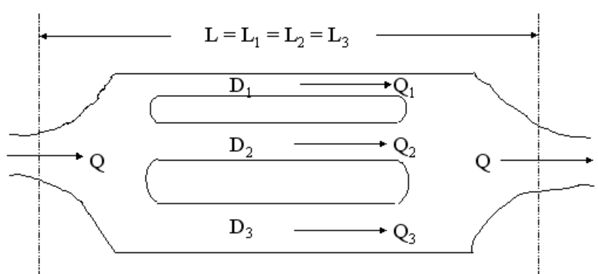


Fig. 7 Pipes in Parallel Arrangement

In this arrangement, the length of the each pipe is same and discharge is distributed in the parallely connected pipes a shown in figure.

Let D1, D2, D3be the diameter of the pipe.
Let L1=L2=L3= L, is length of Pipe connected in parallel
The (hf)1, (hf)2, (hf)3 are the head loss due to friction for each pipe.
The condition for the parallel pipe is
h_f = hf)1= (hf)2= (hf)3
i.e.,

$$h_f = \frac{fL_1V_1^2}{2gD_1} = \frac{fL_2V_2^2}{2gD_2} = \frac{fL_3V_3^2}{2gD_3}$$

Or

$$\frac{8fL_1Q_1^2}{g\pi^2 D_1^5} = \frac{8fL_2Q_2^2}{g\pi^2 D_2^5} = \frac{8fL_3Q_3^2}{g\pi^2 D_3^5}$$

or

$$\frac{Q_1^2}{D_1^5} = \frac{Q_2^2}{D_2^5} = \frac{Q_3^2}{D_3^5}$$

From continuity equation Q= Q1+Q2+Q3

EQUIVIVALENT PIPE

In practice adopting pipes in series may not be feasible due to the fact that

- they may be of not standard size (ie. May not be comemercially available)
- they experience other minor losses.

Hence, the entire system will be replaced by a single pipe of uniform diameter D, of the same length, L=L1+ L2+ L3. This pipe is called as Equivalent Pipe. The below figure shows representation of equivalent pipe.

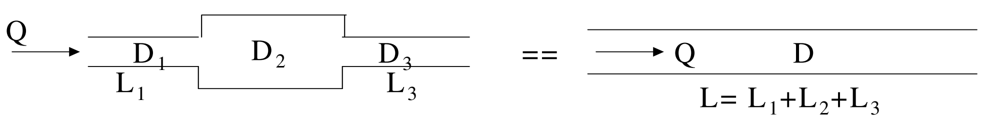


Fig. 8 Equivalent Pipe

In the equivalent pipe system the different diameter will be replaced by a single pipe of uniform diameter D, but of the same length L=L1+ L2+ L3 such that the head loss due to friction for both the pipes, viz equivalent pipe & the compound pipe are the same.

For a compound pipe or pipes in series.

$$h_f = hf_1 + hf_2 + hf_3$$

$$h_f = \frac{8fL_1Q^2}{g\pi^2 D_1^5} + \frac{8fL_2Q^2}{g\pi^2 D_2^5} + \frac{8fL_3Q^2}{g\pi^2 D_3^5} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

for an equivalent pipe,

$$h_f = \frac{8fLQ^2}{g\pi^2 D_1^5} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..8$$

Equating (7) & (8) and simplifying;

$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

Or

$$D = \left\{ \frac{L}{\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}} \right\}^{\frac{1}{5}}$$

PROBLEMS

1. Find the diameter of a Galvanized iron pipe required to carry a flow of 40lps of water, if the loss of head is not to exceed 5m per 1km. Length of pipe is 1km, Assume f=0.02.

Solution:-

D=?, Q=40lps = 40x10-3 m3/s
hf=5m, L=1km = 1000m. f=0.02
Darcy's equation is

$$\therefore D = \left\{ \frac{8fLQ^2}{g\pi^2 h_f} \right\}$$

$$\therefore D = \left\{ \frac{8x0.02x1000x(40x10^{-3})^2}{9.81x\pi^2 x5} \right\}^{\frac{1}{5}}$$

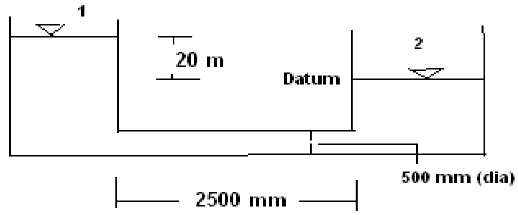
$$D - 0.22m = 220mm$$

Problem-2

Two tanks are connected by a 500mm diameter 2500mm long pipe. Find the rate of flow if the difference in water levels between the tanks is 20m. Take f=0.016. Neglect minor losses.

Solution:-

Applying Bernoulli's equation between (1) & (2) with (2) as datum & considering head loss due to friction hf only,



$$Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_f \; --- (1)$$

Z1 = 20m, Z2 = 0 (Datum);

V1=V2 = 0 (tanks are very large)

p1=p2=0 (atmospheric pressure)

Therefore From (1)

20+0+0 = 0+0+0+hf

Or hf = 20m.

But

$$h_f = \frac{8fLQ^2}{g\pi D^5}$$

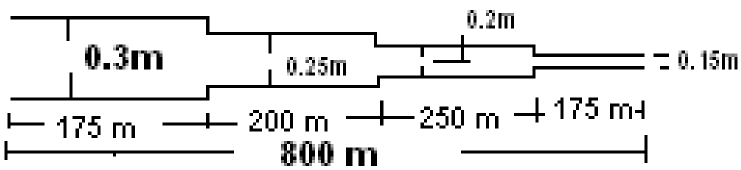
$$Q = \left\{ \frac{20x9.81x\pi^2 x0.5^5}{8x0.016x2500} \right\}^{\frac{1}{2}}$$

$$Q = 0.4348 m^3/\sec = 434.8 lps$$

Problem-3

An existing pipe line 800m long consists of four sizes namely, 30cm for 175m, 25cm dia for the next 200m, 20cm dia for the next 250m and 15cm for the remaining length. Neglecting minor losses, find the diameter of the uniform pipe of 800m. Length to replace the compound pipe.

Solution



L=800m
L1=175m D1=0.3m
L2=200m D2=0.25m
L3=250m D3=0.20m
L4=175m D4=0.15m

For an equivalent pipe

$$\frac{L}{D^5} = \left\{ \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \frac{L_4}{D_4^5} \right\}$$

$$D = \left\{ \frac{L}{\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}} \right\}^{\frac{1}{5}}$$

$$\therefore D = \left\{ \frac{800}{\left(\frac{175}{0.3^5} + \frac{200}{0.25^5} + \frac{250}{0.2^5} + \frac{175}{0.15^5} \right)} \right\}^{\frac{1}{5}}$$

D = Diameter of equivalent pipe = 0.189m less than or equal to 19cm.

Problem-4

Two reservoirs are connected by four pipes laid in parallel, their respective diameters being d, 1.5d, 2.5d and 3.4d respectively. They are all of same length L & have the same friction factors f. Find the discharge through the larger pipes, if the smallest one carries 45lps.

Solution:-

D1=d, D2 =1.5d, D3=2.5d, D4=3.4d

L1=L2=L3=L4= L.

f1=f2=f3=f4=f.

Q1=45x10-3m3/sec, Q2=? Q3=? Q4=?

For pipes in parallel hf1=hf2=hf3=hf4 ,i.e.

$$\frac{Q_1^2}{D_1^5} = \frac{Q_2^2}{D_2^5} = \frac{Q_3^2}{D_3^5} = \frac{Q_4^2}{D_4^5}$$

$$Q_2 = \left\{ \left(\frac{1.5d}{d} \right)^5 x \left(45x10^{-3} \right)^2 \right\}^{\frac{1}{2}} = 0.124 m^3 / \sec$$

$$Q_2 = \left\{ \left(\frac{2.5d}{d} \right)^5 x \left(45x10^{-3} \right)^2 \right\}^{\frac{1}{2}} = 0.4446 m^3 / \sec$$

$$Q_2 = \left\{ \left(\frac{3.4d}{d} \right)^5 x \left(45x10^{-3} \right)^2 \right\}^{\frac{1}{2}} = 0.9592 m^3 / \sec$$

MINOR LOSSES

Minor losses in a pipe flow can be either due to change in magnitude or direction of flow. They can be due to one or more of the following reasons

i)Entry loss
ii)Exit loss
iii)Sudden expansion loss
iv)Sudden contraction loss
v)Losses due to pipe bends and fittings
vi)Losses due to obstruction in pipe.

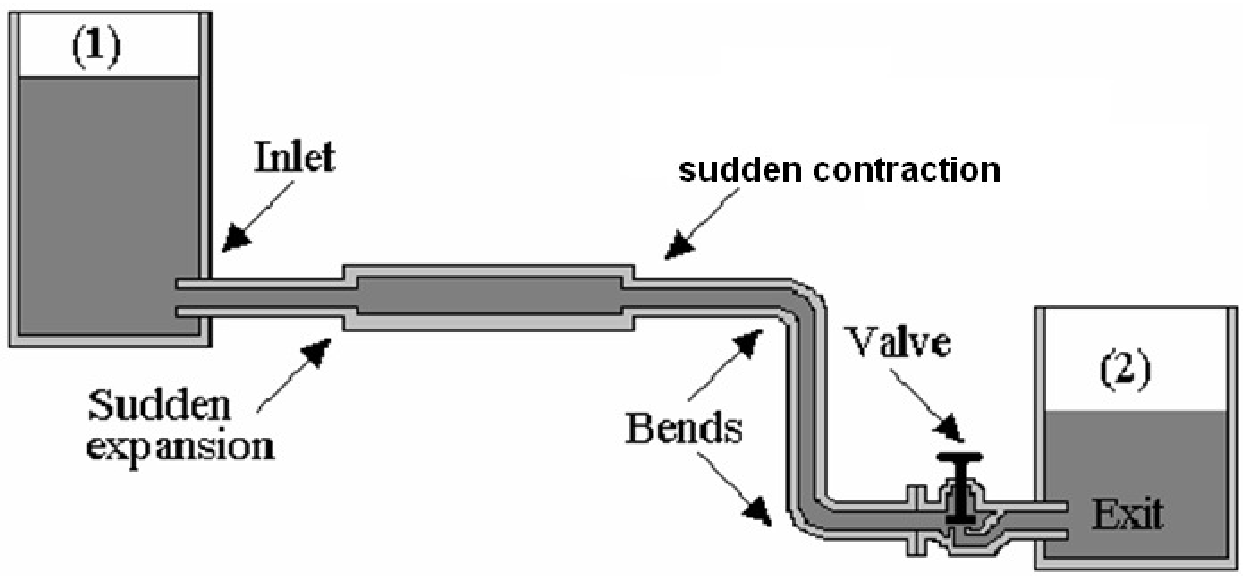


Fig. 8 Minor Losses

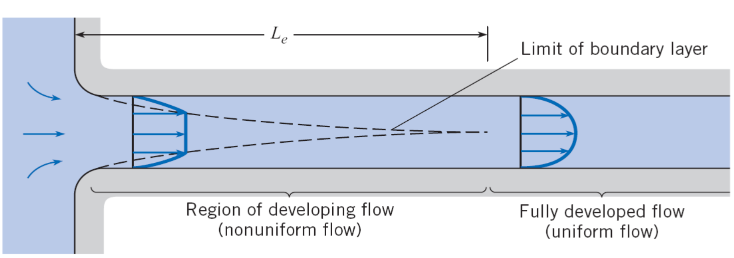
Head loss for inlets, outlets, and fittings will be in the form of:

$$h_L = K\frac{V^2}{2g}$$

where K is a parameter that depends on the geometry.

For a well-rounded inlet, K = 0.1, for abrupt inlet K = 0.5 (much less resistance for rounded inlet).

ENTRANCE LOSSES



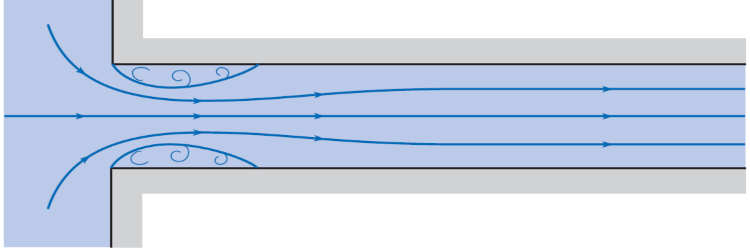


Fig. 9 Enrance Conditions and Losses

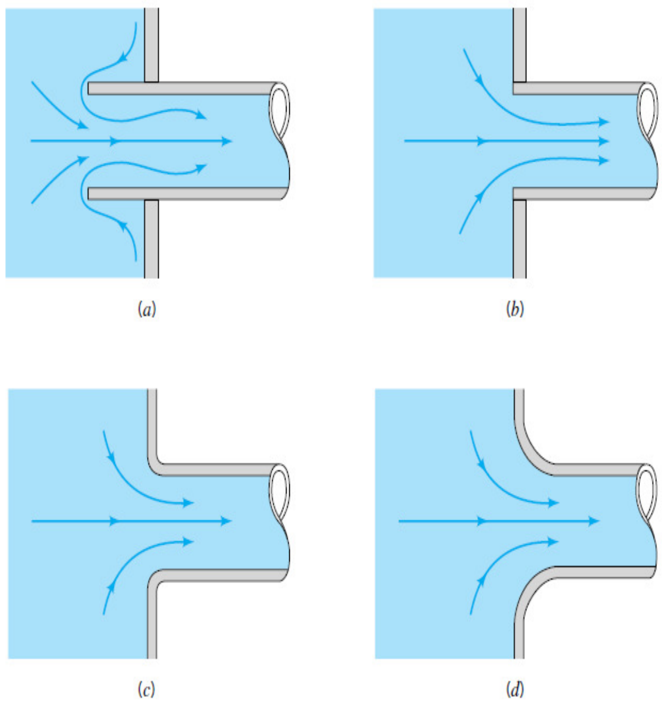


Fig. 10 Entrance flow conditions and loss coefficient (Refs. 28, 29). (*a*) Reentrant, $K_L = 0.8$, (*b*) sharp-edged, $K_L = 0.5$, (*c*) slightly rounded, $K_L = 0.2$ (see Fig. 8.24), (*d*) well-rounded, $K_L = 0.04$ (see Fig. 8.24).

For solving problems : $h_{L_{entry}} = \dfrac{0.5V^2}{2g}$

Exit Loss

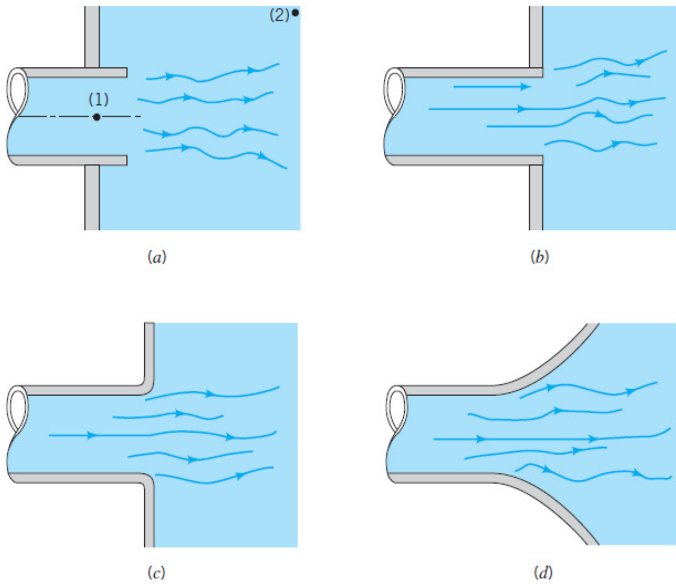
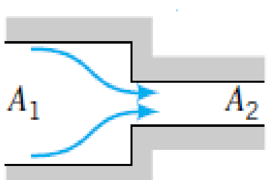


Fig. 10 **Exit flow conditions and loss coefficient. (*a*) Reentrant, $K_L = 1.0$, (*b*) sharp-edged, $K_L = 1.0$, (*c*) slightly rounded, $K_L = 1.0$, (*d*) well-rounded, $K_L = 1.0$.**

For solving problems :

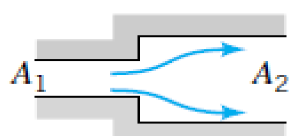
$$h_{L_{exit}} = \frac{V^2}{2g}$$

Losses also occur because of a change in pipe diameter as is shown in Figures. The sharp-edged entrance and exit flows are limiting cases of this type of flow with either $A_1/A_2 = \infty$ or $A_1/A_2 = 0$, respectively.



$$h_L = K_L \frac{V_2^2}{2g}$$

Fig. 11 Sudden Contraction



$$h_L = K_L \frac{V_1^2}{2g}$$

Fig. 12 Sudden Expansion

Loss coefficient for a sudden contraction, $K_L=h_L/(V^2/2g)$, is a function of the area ratio, A_2/A_1,

For solving problems :

$$h_L = 0.5\frac{V_2^2}{2g}$$

Suddden Expansion

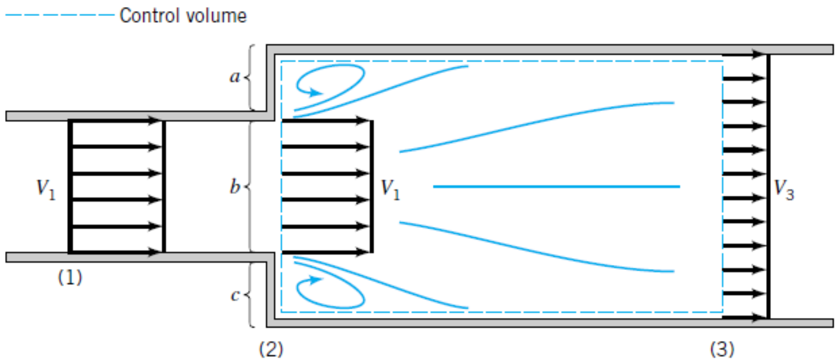


Fig. 13 **Control volume used to calculate the loss coefficient for a sudden expansion.**

Flow in a sudden expansion is similar to exit flow.

Referring Fig., the fluid leaves the smaller pipe and initially forms a jet-type structure as it enters the larger pipe.

Within a few diameters downstream of the expansion, the jet becomes dispersed across the pipe.

In this process of dispersion [between sections (2) and (3)], a portion of the kinetic energy of the fluid is dissipated as a result of viscous effects.

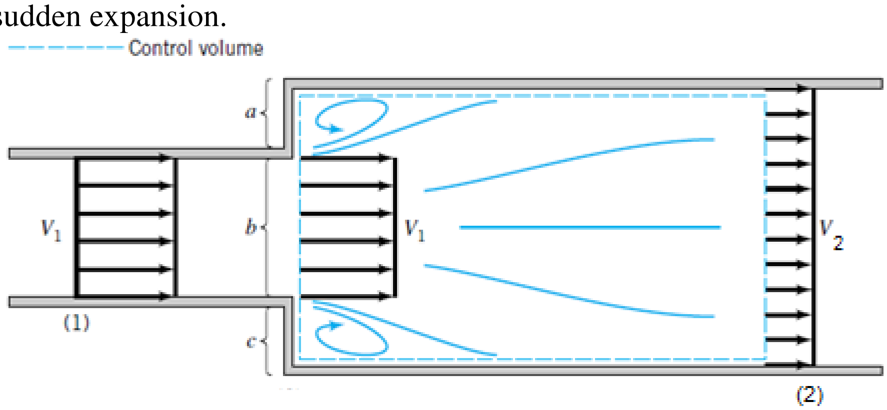
Loss coefficient for sudden expansion can be obtained by means of a simple analysis based on continuity and momentum equations for the control volume shown in figure

Assumption: Flow is uniform at sections (1), (2), and (3) and the pressure is constant across the left-hand side of the control volume ($p_a = p_b = p_c = p_1$)

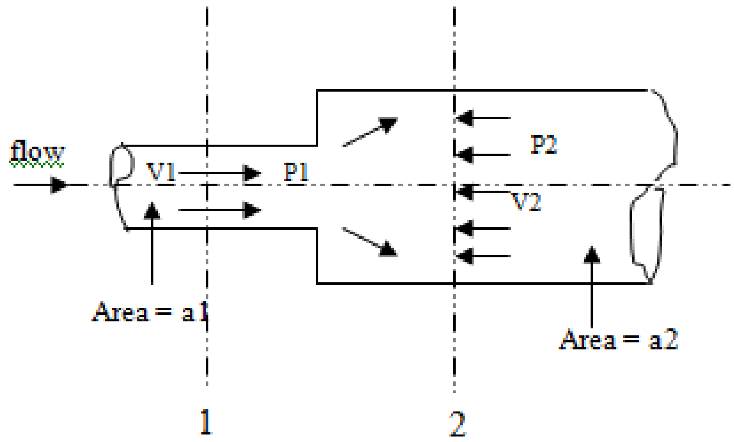
Consider the sections as shown in figure

P1 & P2 are the pressure acting at (1) (1) and (2) (2)

From experiments, it is proved that pressure P1 acts on the area (a2 – a1) i.e. at the point of sudden expansion.



Control volume used to calculate the loss coefficient for a sudden expansion.



V1 and V2 are the velocities.
From II Law of Newton Force = Mass x Acceleration.
The forces acting on the control volume (LHS)

$$\sum forces = +p_1a_1 - p_2a_2 + p_1(a_2 - a_1) --- (i)$$

$$or, \sum forces = a_2(p_1 - p_2) --- (ii)$$

RHS of Neton's second law,
Mass x acceleration = ρ x vol x change in velocity /time

= ρ xvolume/time x change in velocity

$$= \rho x Q x (V_1 - V_2) --- (iii)$$

Substitution (ii) & (iii) in newton's Equation

$$a_2(p_1 - p_2) = pQ(V_1 - V_2)$$

Divide both sides by sp.weight

$$\therefore \left(\frac{p_1 - p_2}{\gamma}\right) = \frac{V_2(V_1 - V_2)}{g} --- (iv)$$

Applying Bernoulli's equation between (1) and (2) with the centre line of the pipe as datum and considering head loss due to sudden expansion hLonly.

$$Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$Z_1 = Z_2$ because pipe is horizontal

$$\therefore \left(\frac{p_1 - p_2}{\gamma}\right) + \left(\frac{V_1^2 - V_2^2}{2g}\right) = h_L \qquad \text{----------- v}$$

Repalcing $(p_1-p_2)/\gamma$ by Eq. (iv) in Eq. (v)

$$h_L = \frac{2V_2(V_1 - V_2) + (V_1^2 - V_2^2)}{2g}$$

$$h_L = \frac{2V_1V_2 - 2V_2^2 + V_1^2 - V_2^2}{2g}$$

$$h_L = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$h_L = \frac{V_2^2 + V_1^2 - 2V_1V_2}{2g}$$

$$h_L = \frac{(V_1 - V_2)^2}{2g} \qquad \text{.....vi}$$

The Equation (vi) represents the loss due to sudden expansion.

Loss of Power

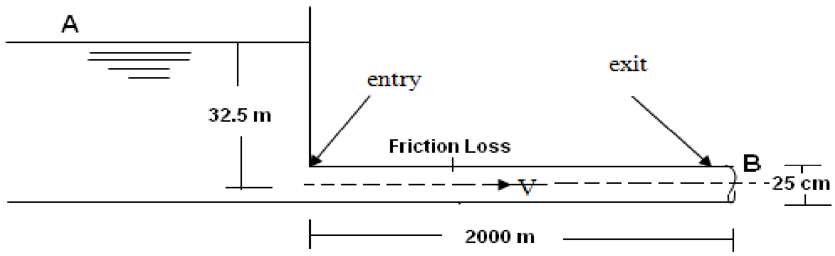
The loss of power in overcoming the head loss in the transmission of fluid is given by

$$P = \gamma Q h_f \; --- (vi)$$

PROBLEMS
Problem-1

A 25cm diameter, 2km long horizontal pipe is connected to a water tank. The pipe discharges freely into atmosphere on the downstream side. The head over the centre line of the pipe is 32.5m, f=0.0185. Find the discharge through the pipe

Solution:



,

Applying Bernoulli's equation between (A) and (B) with (B) as datum & considering all losses.

$$Z_A + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} = Z_B + \frac{p_B}{\gamma} + \frac{v_B^2}{2g} + entryloss + frictionloss + exitloss$$

The tank surface and the outlet are exposed to atmospheric condition and hence, $P_A = P_B$.
When tank area is compared with the pipe area, it is very much grater than the pipe and hence the variation of velocity in the tank can be neglected. Therefore, $V_A = 0$.
The above equation now can be written as,

$$32.5 + 0 + 0 = 0 + 0 + \frac{V^2}{2g} + \frac{0.5V^2}{2g} + \frac{fLV^2}{2gD} + \frac{V^2}{2g}$$

$$32.5 = \frac{V^2}{2g}\left\{1 + 0.5 + \frac{0.0185X2000}{0.25} + 1\right\}$$

$$32.5 = 7.67V^2$$

$$V = 2.06m/s$$

The discharge is calculated using continuity equation.

$$Q = \frac{\pi D^2}{4}V$$

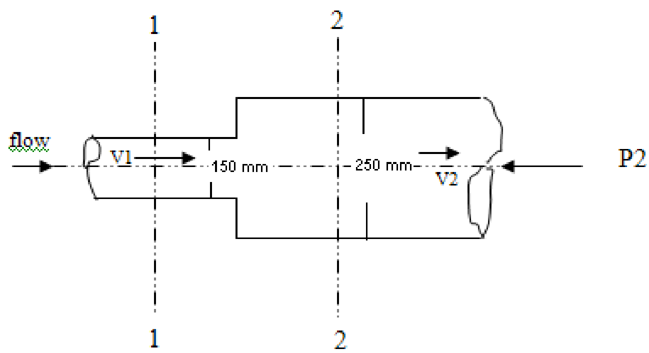
$$\pi x 0.25^2 / 4 \, x2.06 = 0.101 m^3/\sec$$

$$Q = 101lps$$

Problem-2

The discharge through a pipe is 225lps. Find the loss of head when the pipe is suddenly enlarged from 150mm to 250mm diameter.

Solution:
D1=0.15m, D2 = 0.25m Q=225lps = 225m3/sec



Head loss due to sudden expansion is

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

Writing the above equation in terms of discharge,,

$$hL = \left(\frac{4Q}{\pi D_1^2} - \frac{4Q}{\pi D_2^2}\right)^2 X \frac{1}{2g}$$

$$h_L = \frac{16Q^2}{2g\pi^2}\left(\frac{1}{D_1^2} - \frac{1}{D_2^2}\right)^2$$

$$= \frac{16x0.225^2}{2x9.81x\pi^2}\left(\frac{1}{0.15^2} - \frac{1}{0.25^2}\right)^2$$

$$h_L = 3.385m$$

Problem-3

The rate of flow of water through a horizontal pipe is 350lps. The diameter of the pipe is suddenly enlarge from 200mm to 500mm. The pressure intensity in the smaller pipe is 15N/cm^2. Determine (i) loss of head due to sudden enlargement. (ii) pressure intensity in the larger pipe (iii) power lost due to enlargement.

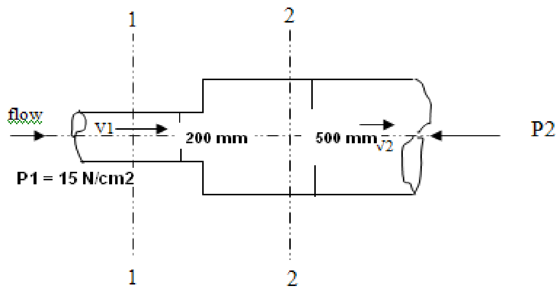
Solution:

Q=350lps=0.35m^3/s

D1=0.2m, D2=0.5m,

P1=15N/cm2

hL=?, p2=?, P=?



From continuity equation

$$V_2 = \frac{4Q}{\pi D_2^2} = \frac{4x0.35}{\pi x0.5^2} = 1.78 m/s$$

$$h_L = \frac{(V_1 - V_2)}{2g} = \frac{(11.14 - 1.78)^2}{2x9.81} = 4.463 mofwater$$

Applying Bernoulli's equation between (1) (1) and (2) (2) with the central line of the pipe as datum and considering head loss due to sudden expansion hL only,

$$Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$0 + \frac{150}{9.81} + \frac{11.14^2}{19.62} = 0 + \frac{p_2}{9.81} + \frac{1.78^2}{19.62} + 4.463$$

$$Z_1 = Z_2 = 0(pipehorizontal)$$

$$p_2 = 166.68 kN/m^2 = 16.67 N/cm^2$$

Power Loss;

$$P = \gamma Q h_L$$

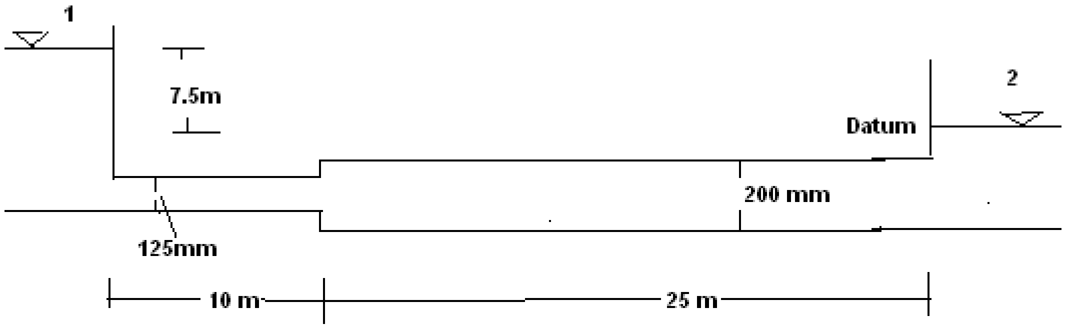
$$= 9.81x0.35x4.463$$

$$P = 15.32 kW$$

Problem-4

Two reservoirs are connected by a pipe line which is 125mm diameter for the first 10m and 200mm in diameter for the remaining 25m. The entrance and exit are sharp and the change of section is sudden. The water surface in the upper reservoir is 7.5m above that in the lower reservoir. Determine the rate of flow, assuming f=0.001 for each of the types.

Solution;



From continuity equation

$$\frac{\pi x 0.125^2}{4} V_1 = \frac{\pi x 0.2^2}{4} V_2$$

$$\therefore V_1 = 2.56 V_2$$

$$Z_A + \frac{p_A}{\gamma} + \frac{V_A^2}{2g} = Z_B + \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + entryloss + frictionloss + sudden\exp ansionloss + frictionloss + exitloss$$

The tank surfaces are exposed to atmospheric condition and hence, $P_1 = P_2$.

When tank area is compared with the pipe area, it is very much grater than the pipe and hence the variation of velocities in the tanks can be neglected. Therefore, $V_1 = V_2 = 0$.

The above equation now can be written as,

$$7.5 + 0 + 0\} = \left\{ 0 + 0 + 0 + \frac{0.5V_1^2}{2g} + \frac{fL_1V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} + \frac{fL_2V_2^2}{2g} + \frac{V_2^2}{2g} \right\}$$

$$7.5 + 0 + 0\} = \left\{ \frac{0.5(2.56V_2^2)}{2g} + \frac{0.01x10x(2.56V_2)^2}{2g} + \frac{(2.56V_2 - V_2)^2}{2g} + \frac{0.01x25x(V_2)^2}{2g} + \frac{V_2^2}{2g} \right\}$$

$147.15 = 3.2768\ (V_2)^2 + 0.65536(V_2)^2 + 2.4336\ (V_2)^2 + 0.25(V_2)^2 + (V_2)^2$

$147.15 = 7.61576\ (V_2)^2$

$V_2 = (147.15/7.61576)^{0.5}$

$V_2 = 4.4$ m/s

Q= ($\pi(0.2)^2/4$)x4.4 = 0.138 m^3/s

WATER HAMMER

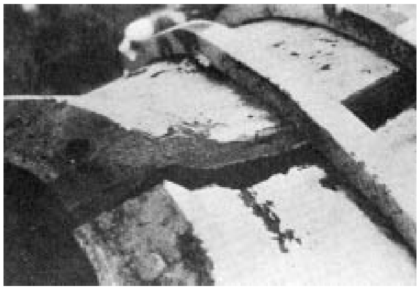
Water Hammer Phenomenon in pipelines

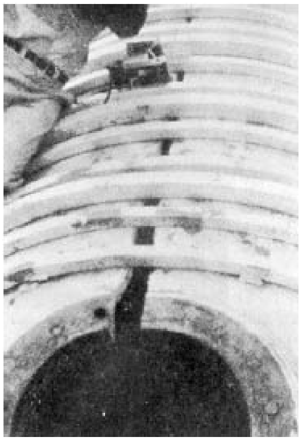
A sudden change of flow rate in a large pipeline (due to valve closure, pump turnoff, etc.) involve a great mass of water moving inside the pipe. The force resulting from changing the speed of the water mass may <u>cause a pressure rise</u> in the pipe with a magnitude several times greater than the normal static pressure in the pipe. This may set up a noises known as **knocking.** This phenomenon is commonly known as the water hammer phenomenon

The excessive pressure <u>may fracture the pipe</u> walls or cause other damage to the pipeline system <u>Has a very high speed</u> **(called celerity, C)** which may reach the speed of sound wave and may <u>create noise</u> called ***knocking***,
Magnitude of this pressure depends on

(i) The mean pipe flow velocity
(ii) The length of the pipe
(iii) The time taken to close the valve and
(iv) The elastic properties of the pipe material and that of water.

Some typical damages





Burst pipe in power sation Big Creek #3, USA

Pipe damage in power station Okigawa

Sudden rise in pressure in the pipe due to the stoppage of the flow generating a high pressure wave, which will have a hammering effect on the walls of the pipe, is known as ***Water Hammer.***

The sudden change of pressure due to a valve closure may be viewed as the result of the force developed in the pipe necessary to stop the flowing water column. The column has a total mass M and is changing its velocity at the rate of dV/dt. According to Newton's second law of motion,

$$F = m\frac{dV}{dt}$$

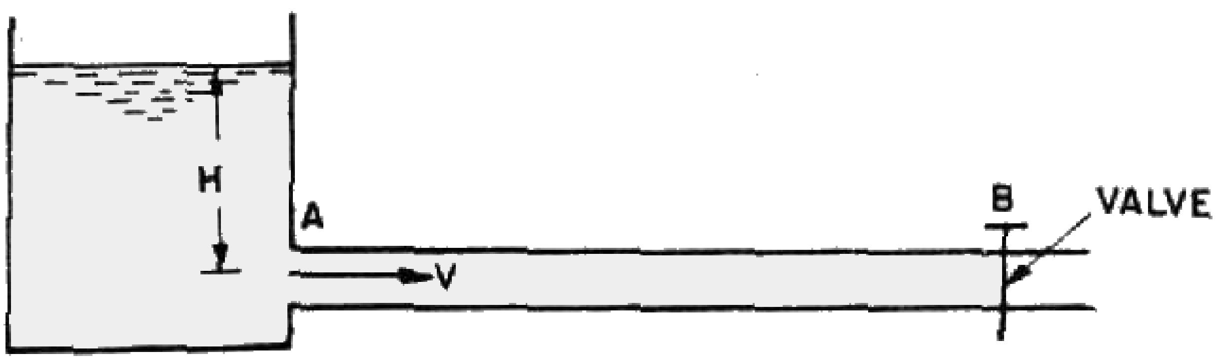
If the velocity of the entire water column could be reduced to zero instantly

$$F = \frac{m(V_0 - 0)}{0} = \frac{mV_0}{0} = \infty$$

The resulting force (hence, pressure) would be infinite. Fortunately, such an instantaneous change is almost impossible because a mechanical valve requires a certain amount of time to complete a closure operation.

Water Hammer Phenomenon in pipelines

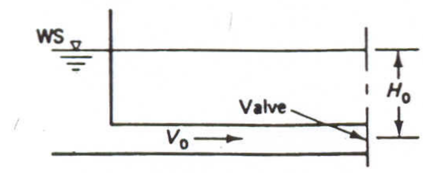
Consider a long pipe *AB*: Connected at one end to a reservoir containing water at a height *H* from the center of the pipe. At the other end of the pipe, a valve to regulate the flow of water is provided.



Kinetic energy of the water moving through the pipe is converted into potential energy stored in the water and the walls of the pipe through the elastic deformation of both.
The water is compressed and the pipe material is stretched.
The following figure illustrates the formation and transition of the pressure wave due to the sudden closure of the valve

Propagation of pressure wave due to valve closure

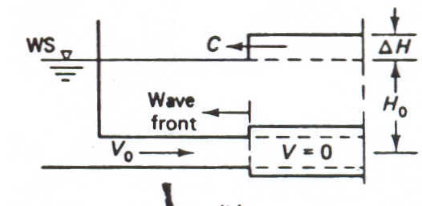
a. Steady state prior to valve closure



(a)

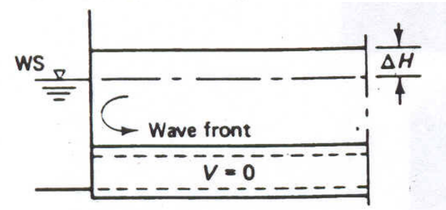
\\

b. Rapid valve closure – pressure increase, pipe walls expand, liquid compression; transient conditions propagate upstream



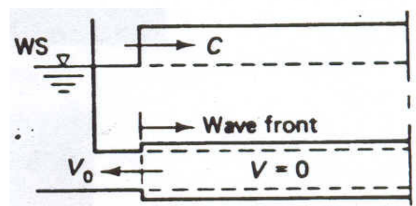
(b)

c. End of step 1 transient process



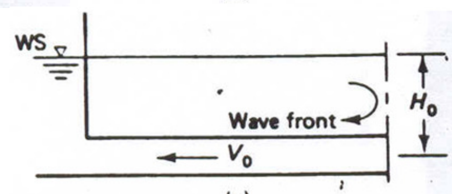
(c)

d. Pipe pressure > tank pressure water flows from pipe to tank relieving pressure in pipe



(d)

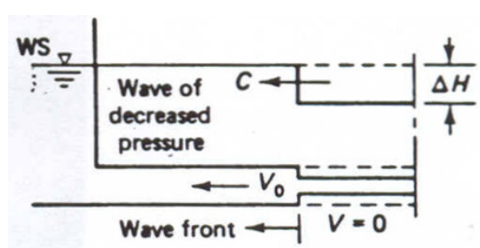
e. Process starts at tank and continues to valve, time L/c, total time 2L/c => water hammer period



(e)

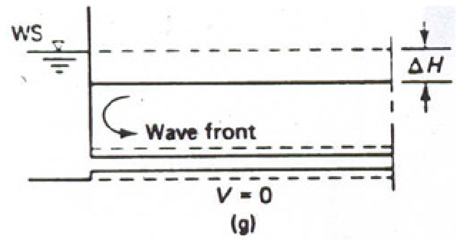
\

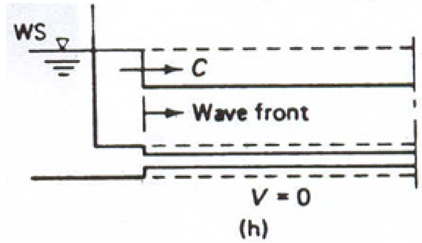
f. Wave of backwater cannot go past the valve, starts wave of negative pressure toward tank



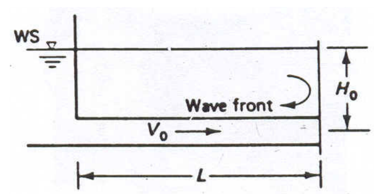
(f)

g & h. Pressure difference causes water to flow toward valve





i. One full cycle, 4L/c



Analysis of Water Hammer Phenomenon

The pressure rise due to water hammer depends upon:
(a) The velocity of the flow of water in pipe,
(b) The length of pipe,
(c) Time taken to close the valve,
(d) Elastic properties of the material of the pipe.
The following cases of water hammer will be considered:
Gradual closure of valve,
Sudden closure of valve and pipe is rigid, and
Sudden closure of valve and pipe is elastic.

The time required for the pressure wave to travel from the valve to the reservoir and back to the valve is:

$$t = \frac{2L}{C}$$

Where:
L = length of the pipe (m)
C = speed of pressure wave, celerity (m/sec)
If the valve time of closure is t_c , then

If $t_c \succ \frac{2L}{C}$ the closure is considered gradual

If $t_c \leq \frac{2L}{C}$ the closure is considered sudden

The speed of pressure wave *"C"* depends on :
the pipe wall material.
the properties of the fluid.
the anchorage method of the pipe.

$$C = \sqrt{\frac{K}{\rho}}$$ if the pipe is rigid

$$C = \sqrt{\frac{E_c}{\rho}}$$ if the pipe is elastic

and

$$\frac{1}{E_c} = \frac{1}{K} + \frac{Dk}{E_p e}$$

Where:
C = velocity (celerity) of pressure wave due to water hammer.
ρ = water density (*1000 kg/m3*).
K = bulk modulus of water ($2.1 \times 10^9\ N/m^2$).
E_c = effective bulk modulus of water in elastic pipe.
E_p = Modulus of elasticity of the pipe material.
e = thickness of pipe wall.
D = diameter of pipe.
k = factor depends on the anchorage method:
= $(\frac{5}{4} - \varepsilon)$ for pipes free to move longitudinally,

= $(1 - \varepsilon^2)$ for pipes anchored at both ends against longitudinal movement

= $(1 - 0.5\varepsilon)$ for pipes with expansion joints.
where ε = poison's ratio of the pipe material (0.25 - 0.35). It may take the value = 0.25 for common pipe materials.

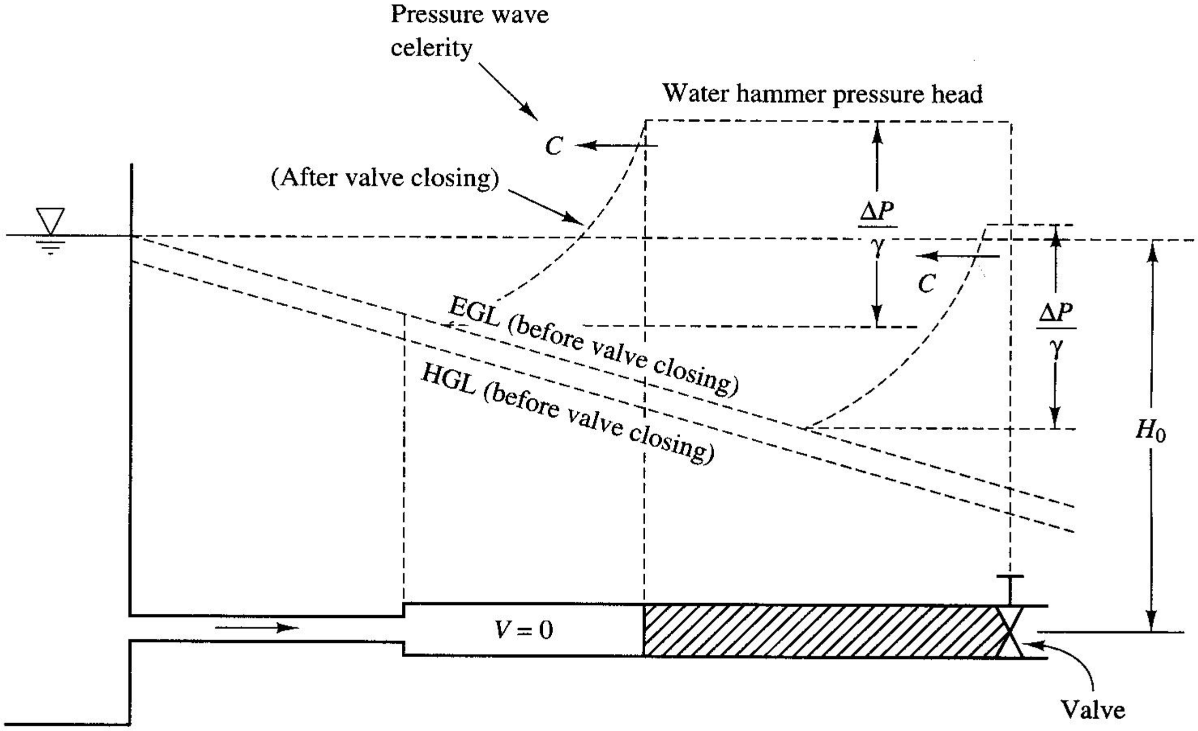
If the longitudinal stress in a pipe can be neglected, $k = 1.0$, and Equation can be simplified

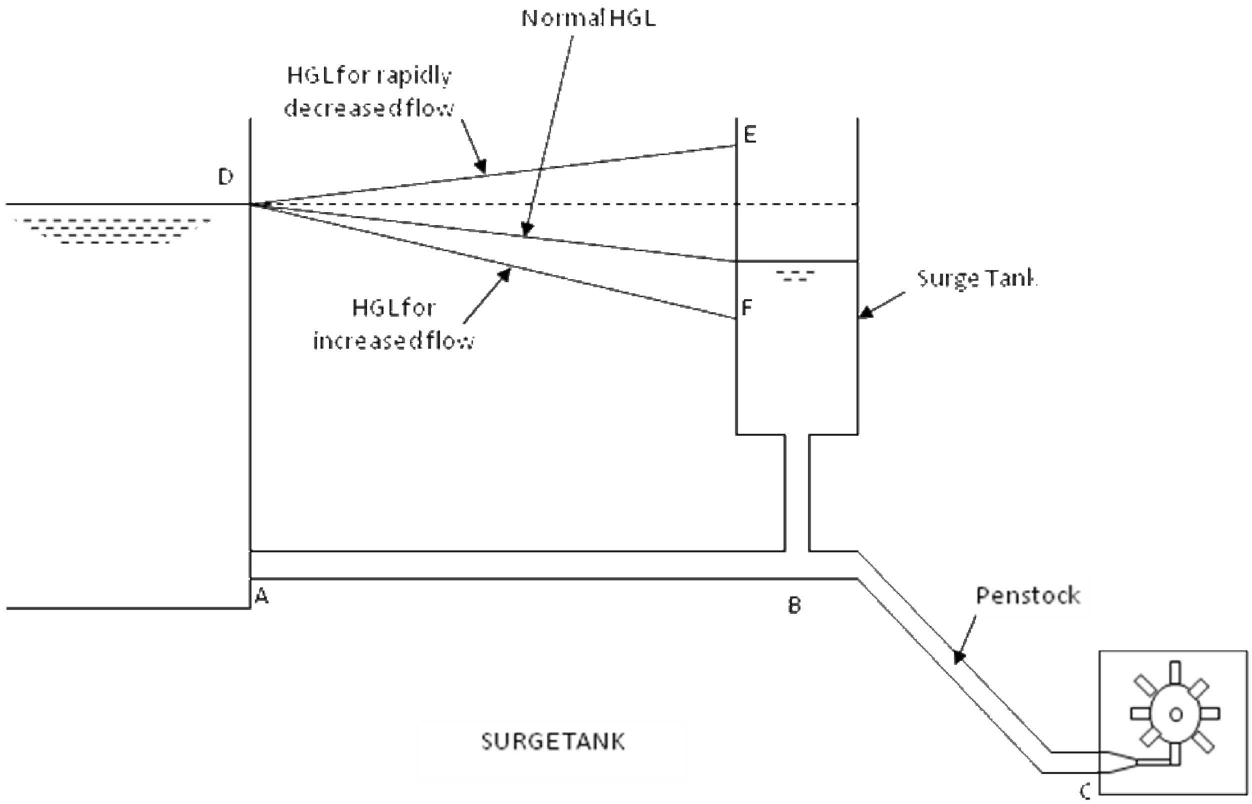
$$\frac{1}{E_c} = \frac{1}{K} + \frac{D}{E_p e}$$

Maximum pressure created by the water hammer

The total pressure experienced by the pipe is

$$P = \Delta P + P_0$$





Instantaneous rise in pressure in a pipe running full due to Gradual closure of valve

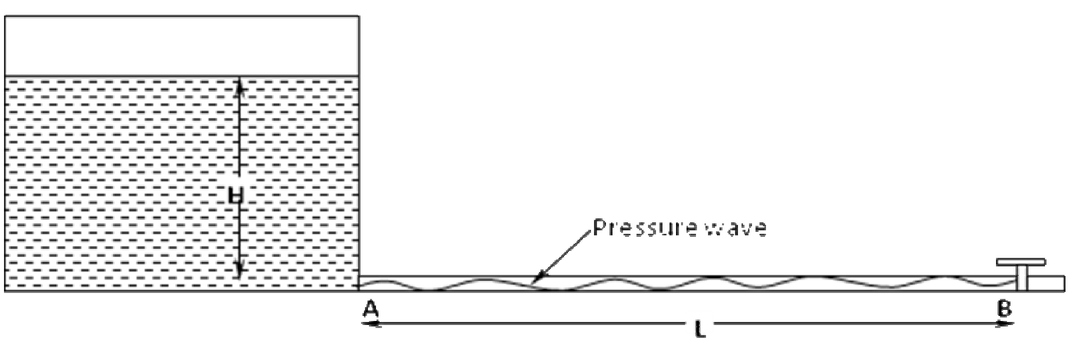
Consider a pipe AB of length ***L*** connected to a tank at A and a valve at B with water flowing in it as shown in Fig.

Let ***V*** be the mean flow velocity

a is the flow cross-sectional area,

p the instantaneous rise in pressure due to gradual closure of valve

t be the actual time of closure of valve.



From Newton's second law of motion,

the retarding force generated against the flow direction is given by the rate of change momentum of the liquid along the direction of the force.

Retardation of water = Change in velocity / Time = $\frac{(V-0)}{t} = \frac{V}{t}$

Retarding force = Mass of water x Retardation = $\rho a L \frac{V}{t}$

The force generated due to pressure wave = Pressure intensity x area = $p_i \times a$ 2

From Eqs. 1 and 2, we get

$$p_i a = \rho a L \frac{V}{t}$$

Hence the increase in pressure rise due to gradual closure of valve:

$$\Delta p \text{ or } \quad p_i = \frac{\rho L V}{t}$$

where,
V = initial velocity of water flowing in the pipe before pipe closure
t = time of closure.
L = length of pipe.
ρ = water density.

The pressure head caused by the water hammer is

$$H = \frac{p_i}{\rho g} = \frac{\rho L V}{\rho g t} = \frac{L V}{g t}$$

OR

$$\Delta H = \frac{\Delta P}{\gamma} = \frac{\rho L V}{\rho g t} = \frac{L V}{g t}$$

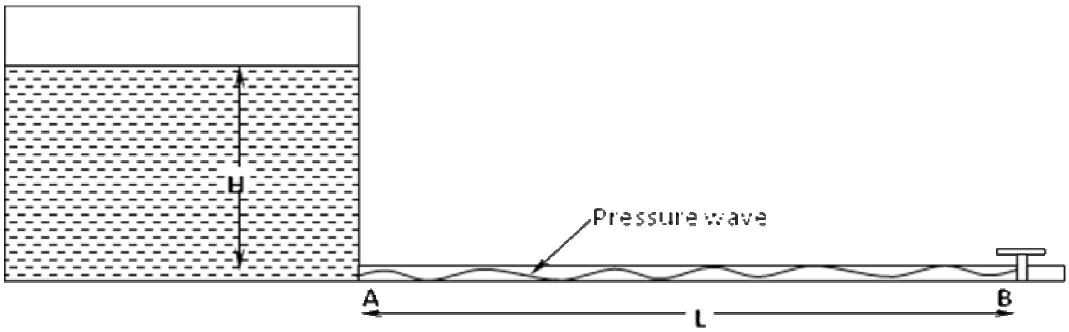
Instantaneous rise in pressure in a pipe running full due to Sudden closure of valve when the pipe is rigid $t_c \leq \frac{2L}{C}$

When the valve provided at the downstream end is closed suddenly and the pipe is rigid, then the converted pressure energy from the kinetic energy due to closure is to be absorbed by the fluid due to its compressibility only.

i.e. $\quad \boldsymbol{E_k = E_w}$ 1

{Pressure Energy converted from Kinetic energy} = {Pressure energy absorbed by water due to its compressibility}

Consider the pipe AB of length ***L***
cross-sectional area ***a***
water of mass density ***ρ,***
weight density ***γ***
bulk density of water ***K***
mean velocity of flow ***V***
be suddenly stopped due to closure of valve provided at B.



The kinetic energy of flowing water before closure of valve will be converted to strain energy, when the effect of friction and elasticity of pipe material are ignored.

Loss of kinetic energy E_k

$$= \frac{1}{2} \times \text{mass of water} \times V^2$$

As mass = ρ **x volume =** ρ **x** aL

Loss of kinetic energy, $E_k = \frac{1}{2} \times \rho a L \times V^2$ …..2

Gain in strain energy $E_w = \frac{1}{2}\left(\frac{p_i^2}{K}\right)\times \text{Volume} = \frac{1}{2}\left(\frac{p_i^2}{K}\right)\times aL$ ………..3

From Eqs. 2 and 3, we get

$$\rho a L V^2 = \left(\frac{p_i^2}{K}\right)\times aL$$

OR

$$p_i^2 = \rho K V^2$$

OR

$$p_i = V\sqrt{\rho K}$$

But Celerity $C = \sqrt{\frac{K}{\rho}}$

Substituting for the value of ***C*** in the above equation for pressure rise, we get

$$\Delta p \text{ or } p_i = \rho V C$$

$$\Delta H = \frac{CV}{g}$$

But for rigid pipe $C = \sqrt{\frac{K}{\rho}}$ so $\Delta H = \frac{V}{g}\sqrt{\frac{K}{\rho}}$

*Instantane*ous rise in pressure in a pipe running full due to Sudden closure of valve when the pipe is elastic

When the valve provided at the downstream end is closed suddenly and the pipe is elastic, then the converted pressure energy from the kinetic energy due to the valve closure is to be absorbed by both the fluid due to its compressibility and the elasticity of the pipe.

i.e. $\boldsymbol{E_k = E_w + E_p}$ ……………1

{Pressure Energy converted from Kinetic energy} = {Pressure energy absorbed by water due to its compressibility} + {Pressure energy absorbed by the Elastic pipe due to its expansion}

E_k and E_w can be computed as in the previous derivation.

$E_k = \frac{1}{2} \times \rho\, a\, L \times V^2$ ……………2

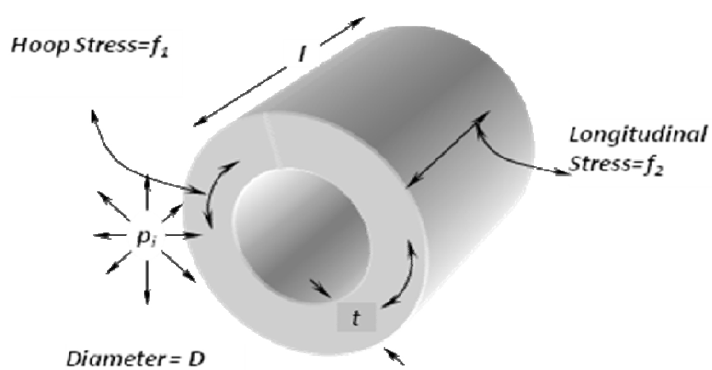
$$E_w = \frac{1}{2}\left(\frac{p_i^2}{K}\right) \times \text{Volume} = \frac{1}{2}\left(\frac{p_i^2}{K}\right) \times aL$$

Computation of $\boldsymbol{E_p}$ can be done by simulating the situation to the thick cylinder subjected to internal fluid pressure.

Let ***t*** be the thickness of the elastic pipe wall and assume that it is small compared to its diameter ***D.***

Let $\boldsymbol{f_1}$ be the hoop or circumferential and stress

$\boldsymbol{f_2}$ be the longitudinal stress as shown in figure.



Let the Young's modulus of the pipe material be ***E*** and poisons ratio ***1/m***

Let the instantaneous fluid pressure be $\boldsymbol{p_i}$.

From the knowledge oaf Strength of materials, we can write that

$f_1 = \frac{p_i D}{2t}$ and $f_2 = \frac{p_i D}{4t}$ Hence $\boldsymbol{f_1 = 2 f_2}$

Further, the **strain energy** stored in pipe per unit volume is given by

$$E_p / V_1 = \frac{1}{2E}\left[f_1^2 + f_2^2 - \frac{2 f_1 f_2}{m} \right]$$

Substituting $f_1 = 2 f_2$, we get

$$E_p / V_1 = \frac{1}{2E}\left[4 f_2^2 + f_2^2 - \frac{4 f_2^2}{m} \right]$$

$$E_p / V_1 = \frac{f_2^2}{2E}\left[5 - \frac{4}{m} \right]$$

Substituting for f_2, and $V_1 = \pi D t l$ we get

$$E_p = \frac{p_i^2 D^2}{16 t^2} \frac{1}{2E}\left[5 - \frac{4}{m} \right] \pi D t l \qquad \text{……………..4}$$

Substituting Eqs. 2,3 and 4 in Eq. 1,

$$\frac{1}{2} \times \rho a L \times V^2 = \frac{1}{2}\left(\frac{p_i^2}{K} \right) \times aL + \frac{p_i^2 D^2}{16 t^2} \frac{1}{2E}\left[5 - \frac{4}{m} \right] \pi D t L$$

Simplifying, we get

$$\frac{1}{2} \times \rho a L \times V^2 = \frac{1}{2}\left(\frac{p_i^2}{K} \right) \times aL + \frac{p_i^2 D}{4t} \frac{1}{2E}\left[5 - \frac{4}{m} \right] \frac{\pi D^2}{4} L$$

But $a = \frac{\pi D^2}{4}$ and aL/2 gets canceled on both sides, now the equation takes the form as,

$$\rho V^2 = \left(\frac{p_i^2}{K} \right) + \frac{p_i^2 D}{4t} \frac{1}{E}\left[5 - \frac{4}{m} \right] = p_i^2 \left[\left(\frac{1}{K} \right) + \frac{D}{4tE}\left(5 - \frac{4}{m} \right) \right]$$

Solving for p_i ;

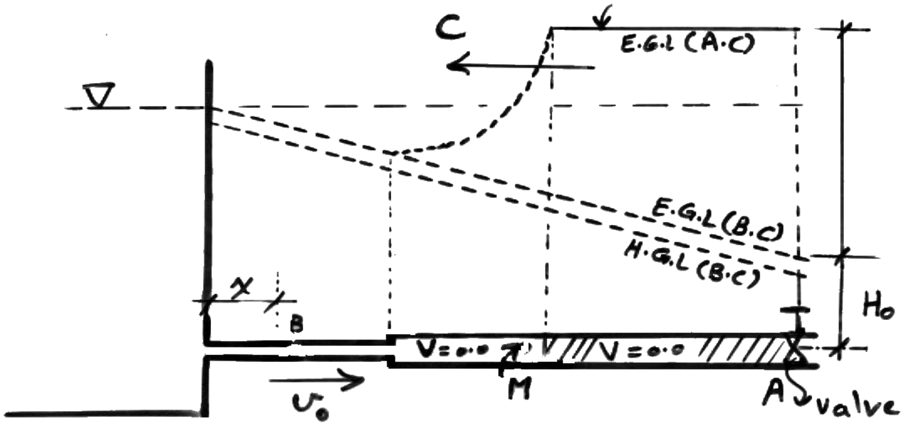
$$p_i = V \sqrt{\frac{\rho}{\left[\left(\frac{1}{K} \right) + \frac{D}{4tE}\left(5 - \frac{4}{m} \right) \right]}} = V \sqrt{\frac{\rho}{\left[\left(\frac{1}{K} \right) + \frac{D}{tE}\left(\frac{5}{4} - \frac{1}{m} \right) \right]}}$$

The above expression gives the instantaneous rise in pressure in an elastic pipe due to sudden closure of Valve.

If the Poisons ration is not given, it can be assumed as ¼. Then Eq. 5reduces to

$$p_i = V \sqrt{\frac{\rho}{\left[\frac{1}{K} + \frac{D}{tE} \right]}} \qquad \text{………………6}$$

Applying the water hammer formulas we can determine the energy gradient line and the hydraulic gradient line for the pipe system under steady flow condition.



So the total pressure at any point *M* after closure (water hammer) *is*

$$P_M = P_{M,beforeclosure} + \Delta P$$

Or

$$H_M = H_{M,beforeclosure} + \Delta H$$

PROBLEM

A steel pipe 1524 m long laid on a uniform slope has an 45.72 cm, diameter and a 5.08 cm wall thickness. The pipe carries water from a reservoir and discharges into the air at an elevation 45.72 m below the reservoir free surface. A valve installed at the downstream end of the pipe allows a flow rate of 0.708 m3/s. If the valve is completely closed in 1.4 sec, calculate the maximum water hammer pressure at the valve. Neglect the longitudinal stress.

Take K= 2.07 x 10^{11} N/m^2, E_p = 1.93 x 10^{11} N/m^2

Solution:

Effective bulk modulus of water In elastic pipe, E_c , is given by

$$\frac{1}{E_c} = \frac{1}{K} + \frac{D}{E_p e}$$

1/E_c =(1/ 2.07 x 10^{11}) + (0.4572/(1.93 x 10^{11} x0.0508))

Hence

E_c = 1.888 x 10^9 N/m^2

$C = \sqrt{\frac{E_c}{\rho}}$ = (1.888 x 10^9 /1000)$^{0.5}$ = **1374.045m/s**

The required for the wave to return to the value, t_c = (2L/C)

t_c = (2x 1524/1374.045) = 2.22 sec

But the time of closure of valve = 1.4 sec

There fore, $t < t_c$. Hence it is a case of instantaneous closure.
Since the longitudinal stresses are neglected the case is of rigid pipe.
The instataneous increase in pressure is given by $p_i = \rho V C$.
The velocity, v= Q/A = 0.708/(π x(0.4572)^2/4) = 4.312 m/s.
Therefore,maximum water hammer pressure = 1000x4.312x1374.045 = 59235508 N/m^2